



2011 Massachusetts Curriculum Framework for Mathematics

*Incorporating the Common Core State Standards
for Mathematics*

**Massachusetts Department of Elementary and
Secondary Education and
the Massachusetts Readiness Centers
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Goals for this Session

This presentation will...

- Provide background on the development of the 2011 MA Curriculum Framework for Mathematics
- Show how the new framework is organized
- Point to some key changes in the new framework
- Highlight improvements- increased **focus**, **coherence**, **clarity**, and **rigor**
- Engage you in a “dive” activity into the framework



Purpose of the Standards

“These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms.”

-2011 Massachusetts Curriculum Framework for Mathematics (page 14)

-Common Core State Standards for Mathematics (page 5)



Supporting changes in practice

- The new standards support improved curriculum and instruction due to increased:
 - **FOCUS**, via critical areas at each grade level
 - **COHERENCE**, through carefully developed connections within and across grades
 - **CLARITY**, with precisely worded standards that cannot be treated as a checklist
 - **RIGOR**, including a focus on College and Career Readiness and Standards for Mathematical Practice throughout Pre-K-12



The Role of Massachusetts in Developing the Mathematics Common Core State Standards

ESE curriculum and assessment staff:

- Served on the working teams developing the standards
- Formally submitted written comments
- Engaged MA teachers, teacher educators, mathematics faculty, and researchers on external review and validation teams



Evidence Base for the Standards

- Standards from high-performing countries, leading states, and nationally-regarded frameworks, such as the American Diploma Project and NCTM Math Focal Points
- National Assessment of Educational Progress (NAEP) Frameworks, international assessments (e.g., TIMSS and PISA) and longitudinal NAEP, SAT, and ACT scores
- Lists of works consulted and research base are included in the Massachusetts Mathematics Curriculum Framework.



Adding Pre-K Standards to the K-12 Common Core

- EEC/ESE staff, experts in early childhood education drafted Pre-kindergarten standards based on
 - The Kindergarten Common Core Standards (2010)
 - The Massachusetts Guidelines for Preschool Learning Experiences (2003)
 - The Massachusetts Kindergarten Learning Experiences (2008)
 - Draft Massachusetts Pre-K standards created by Curriculum Framework Revision panels (2007-2010)
 - Draft Massachusetts Standards for Infants and Toddlers (2010)



Adding MA Standards to the K-12 Common Core

- MA added about 4% additional standards:
 - 13 K-8 additions
 - No additions in Kindergarten, grade 3 or grade 8
 - One addition in grade 4 and grade 5
 - Two additions in grade 1, grade 2, and grade 7
 - Five additions in grade 6
 - 9 high school additional standards
 - Included in conceptual categories: Number and Quantity, Algebra, Functions, and Geometry
- Example of additions: introduction of coins in gr.1; concept of negative numbers in grade 5; measurement precision in high school



2011 MA Curriculum Framework for Mathematics Organization

- Introduction (pg.7)
- Guiding Principles for Mathematics Programs (pg.9)
- Standards for Mathematical Practice (pg.15)
- Pre-K to 8 Grade-level standards (pg.18-65)
 - Grade-level Introductions highlighting critical areas
 - Grade-level Overviews of the domains and clusters
- High School Standards: Conceptual Categories (pg.66-93)
- High School Model Pathways and Courses (pg.94-151)
- Appendices (pg.152-155)
- Sample of work consulted (pg.156-159)
- Glossary (pg.160-167)
- Tables (pg.168-171)

(8) Pre-K-12 Standards for Mathematical Practice

“Expertise” for students at all grade levels:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Grade Level Introduction

Grade Level Focus

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeters and inches) and use measurement tools with the understanding that linear measure involves an iteration of a unit. The smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Critical Area

| | | | |
|-------|-----------|---------|-------|
| Focus | Coherence | Clarity | Rigor |
|-------|-----------|---------|-------|

Grade Level Overview ex.

Grade 5 Overview

Operations and Algebraic Thinking

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

The Number System

- Gain familiarity with concepts of positive and negative integers

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
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Format of Pre-K-8 Standards

Domain

Grade Level
2.NBT (code)

Number and Operations in Base Ten 2.NBT

Cluster

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
 - a. 100 can be thought of as a bundle of ten tens — called a “hundred.”
 - b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, and nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.

Standard
2.NBT.1 (code)

Cluster

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value, properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand

Focus

Coherence

Clarity

Rigor



Organized by Domains Rather than Strands

| Pre-K-8 Domains Progression | | | | | | | | | | |
|---------------------------------------|----|---|---|---|---|---|----|---|---|---|
| Domains | PK | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Counting and Cardinality | MA | | | | | | | | | |
| Operations and Algebraic Thinking | MA | | | | | | | | | |
| Number and Operations in Base Ten | | | | | | | | | | |
| Number and Operations - Fractions | | | | | | | | | | |
| Ratios and Proportional Relationships | | | | | | | | | | |
| The Number System | | | | | | | MA | | | |
| Expressions and Equations | | | | | | | | | | |
| Functions | | | | | | | | | | |
| Geometry | MA | | | | | | | | | |
| Measurement and Data | MA | | | | | | | | | |
| Statistics and Probability | | | | | | | | | | |

Ex. of Specificity in 2011 Standards

Former Framework:

MA.4.N.5 **Identify and generate** equivalent forms of common decimals and fractions less than one whole.

New Framework:

4.NF.1 Explain why a fraction a/b is equivalent to fraction $(nxa)/(nxb)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to **recognize and generate** equivalent fractions.



Pre-K-8 Standards Progression Provides a Strong Foundation for Algebra

- Focus on place value, operations, and fractions in early grades
- Increased attention to proportionality, probability and statistics in middle grades
- In depth study of linearity and introduction of functions in Grade 8



High School Organization: Conceptual Categories, grades 9-12

- Number and Quantity (N)
- Algebra (A)
- Functions (F)
- Geometry (G)
- Modeling (★)
- Statistics and Probability (S)

High School Standards

- Conceptual Categories
 - Cross course boundaries
 - Span high school years
- Standards
 - “Core” for common mathematics curriculum for all students to be college and career ready
 - “College Ready” for entry level credit bearing course
 - (+) Additional mathematics that students should learn in order to take courses such as calculus, discrete mathematics, or advanced statistics.

Format of High School Standards

Algebra

Conceptual Category

Domain

Seeing Structure in Expressions

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
 - a. Interpret parts of an expression, such as terms, factors, and coefficients.
 - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $x^2 + 9x - 14$ as $(x + 3)^2 - 5(x + 2) - 14$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
 - a. Factor a quadratic expression to reveal the zeros of the function it defines.
 - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
 - c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Code

A-SSE

Modeling Symbol

Standard
A.SSE.2

Focus

Coherence

Clarity

Rigor

High School Pathways

Two model course pathways

➤ Traditional:

✓ Algebra I, Geometry, Algebra II

➤ Integrated:

✓ Mathematics I, Mathematics II, Mathematics III

Both pathways address the same standards and prepare students for additional courses such as:

✓ Precalculus, Advanced Quantitative Reasoning



Critical Areas
bring FOCUS
to the New Standards

Activity



Desired Outcomes

Participants will

- Become familiar with the fourth grade *Critical Areas*.
- Understand how the *Critical Areas* help organize and bring focus to the fourth grade standards.



Critical Areas

- There are two to four critical areas for instruction in the introduction **for each grade level, model course or integrated pathway.**
- They bring **focus** to the standards at each grade by providing the big ideas that educators can use to build their curriculum and to guide instruction.



Investigating FOCUS (in 30 minutes)

- In teams of 3, each person selects one of the Grade 4 Critical Areas on p. 38.
- Read your Critical Area and underline the key words that help summarize this area. (3 min.)
- On your recording sheet, indicate which standards from pages 40-42 seem to fall with in your Critical Area. (5-10 min.)
- In your team, have each person share the key words for their area and one interesting insight. (6 min.)
- As a team, discuss how Critical Areas can help organize and bring focus to the grade level standards. (5 min.)



Share and report out. (5 min.)

Initial Activities to be posted

- Drafted and to be posted soon:
 - **FOCUS** - Classify standards within Critical Areas
 - **COHERENCE** - Look at how clusters relate to each other within and across grade levels
 - **CLARITY** - Use the crosswalk to compare the new and former standards and think about implications for instruction
 - **RIGOR** - Identify which standards lend themselves to which Mathematics Practices

Some of the National Projects Underway....

- PARCC Model Content Framework project
 - Scope and Sequence for each grade
 - Narratives to help unwrap the standards
- National Council for Supervisors of Mathematics
 - Illustrating the standards for mathematical practice PD materials
- Gates Foundation, (<http://illustrativemathematics.org/>) led by the original standards writers
 - Illustrative Mathematics Project will produce a complete set of sample problems and tasks illustrating the standards.
- CCSSO, Bill Bush
 - Tool for analyzing instructional materials



Continuing Updates

- The 2011 Frameworks and side-by-side comparisons are available at <http://www.doe.mass.edu/candi/commoncore>
- Please check this site regularly for additional resources and updates on professional development.