Resource Guide to the 2011 Massachusetts Curriculum Frameworks

for Students with Disabilities

MATHEMATICS

Pre-Kindergarten–Grade 12

(Incorporating the Common Core State Standards)

Fall 2014

This document was prepared by the
Massachusetts Department of Elementary and Secondary Education

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Commissioner

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Introduction

The 2014 version of the *Resource Guide to the 2011 Massachusetts Curriculum Frameworks for Students with Disabilities* ("the Resource Guide") incorporates the Common Core State Standards for Mathematics, plus a small number of unique standards approved for Massachusetts students, in an instructional guide intended for students with disabilities who have not yet mastered the performance expectations for typical students in their grade.

The 2014 Resource Guide is intended to ensure that all students receive instruction in the Common Core State Standards (plus the unique Massachusetts standards) at levels that are challenging and attainable for each student. It is also intended to serve as a guide for teachers who work with students with more significant disabilities who are participating in the MCAS alternate assessment.

Students with disabilities are expected to achieve the same standards as their nondisabled peers. However, they may need to learn the necessary knowledge and skills differently, including their presentation at lower levels of complexity, in smaller segments, and at a slower pace. While the standards define the concepts, skills, and content that should be taught and learned by all students in each grade, the 2014 Resource Guide identifies "entry points" for each standard that allow educators to teach standards-based skills that encourage students to approach the grade-level standard. It is especially suited for instructing students with significant disabilities who take the MCAS Alternate Assessment (MCAS-Alt) because it aligns less complex skills and content with grade-level subject matter, and allows students to progress along a continuum as they approach grade-level complexity.

Purpose of the Resource Guide

This guide is intended for use by educators to align and develop instruction based on the 2011 Massachusetts Curriculum Frameworks for students with disabilities, particularly those who cannot, in the judgment of their IEP team, or based on a 504 plan, participate in standard MCAS tests even with the use of test accommodations. These students are required to participate in the MCAS-Alt and to maintain a structured collection of evidence for a student portfolio that will be submitted to the state each spring. The entry points described in this guide provide students with disabilities access to the same standards as non-disabled students, as required by law. This Resource Guide replaces the Mathematics section of 2006 Resource Guide for students in grades preK–12.

MCAS-Alt Transition to the 2011 Massachusetts Curriculum Frameworks

Students in grades 3–10 who are taking the 2014 MCAS-Alt will be assessed on the **2011 Massachusetts Curriculum Framework—Mathematics** standards listed in this Resource Guide, according to the schedule shown in Table 1.

Students in grades 10-12 who are taking the 2014 MCAS-Alt for the purpose of meeting the state's Competency Determination requirement in order to be eligible for a high school diploma will continue to be assessed on the November **2000 Curriculum Framework—Mathematics** standards for grade 10 at least through the graduating class of 2018.

Resource Guides for four subjects are available at http://www.doe.mass.edu/mcas/alt/resources.html.

How to Use this Resource Guide

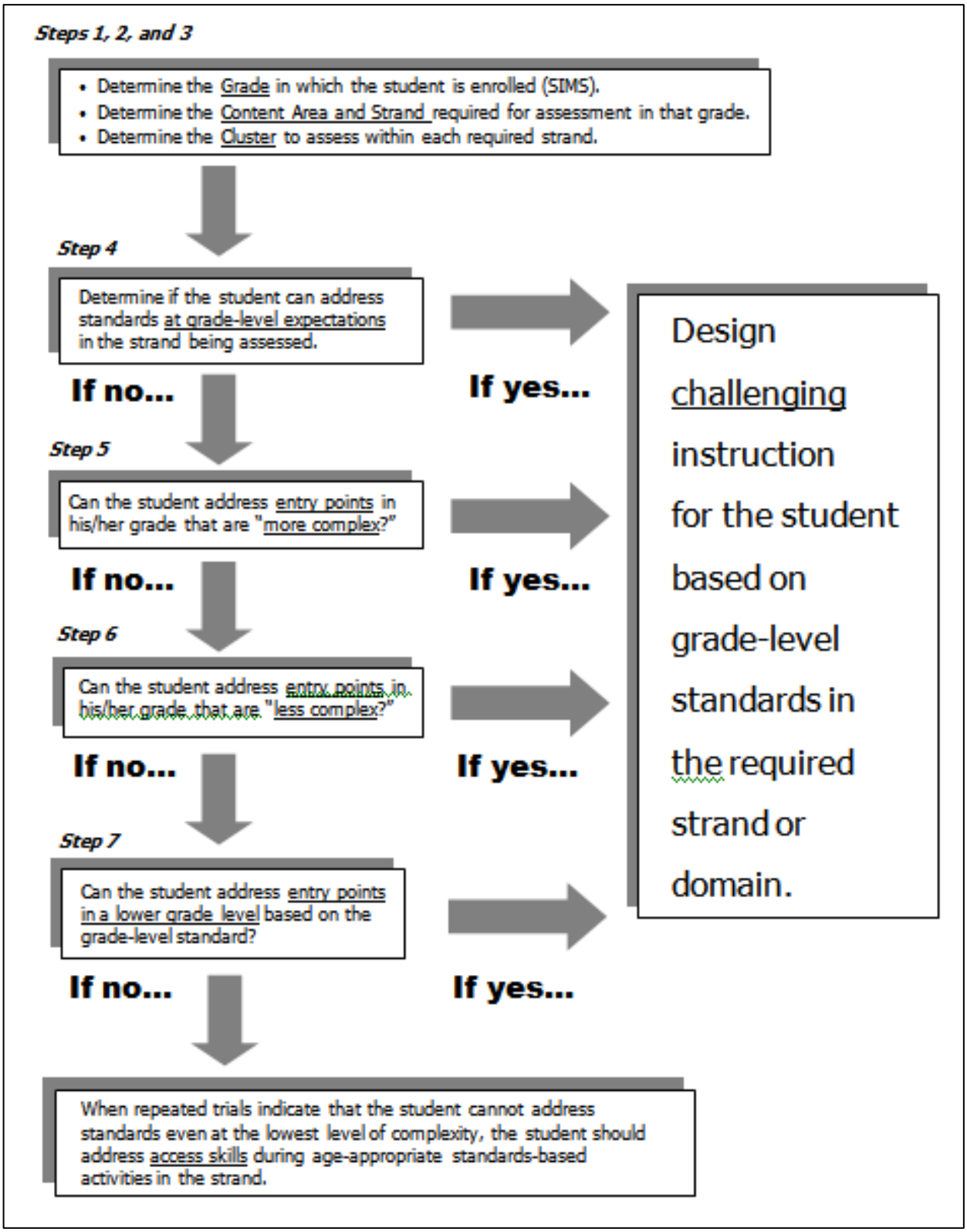
Educators should refer to the standards listed at the grade level of the student for guidance in identifying standards-based instructional goals for students with disabilities, since goals should be based on the general academic curriculum learned by all students. Educators should refer to the "entry points" listed in the Resource Guide to set realistic, yet challenging, measurable outcomes for students with disabilities that are aligned with grade-level standards, but at lower levels of complexity. Figure 1 will assist educators in identifying the appropriate level of complexity of entry points for each student. When used in conjunction with the Guidelines for Collecting Data on Student Performance section of the *Educator's Manual for MCAS-Alt*, student progress can be documented and instruction optimized.

Organization of the Pre-Kindergarten through High School Mathematic Standards

In the Fall 2014 Resource Guide, the content area of Mathematics for students in **preK through grade 8** is organized into **domains** for the successive grades in which that domain is taught. For students in **high school**, Mathematics is organized into Conceptual Categories. Domains and conceptual categories each contain a set of **anchor standards**, followed by **grade-specific standards** that are coded by grade level and domain. **Standards** in each grade are grouped into **clusters,** or subsets of related standards, within each domain or conceptual category.

On the pages following the list of cluster headings and standards, **entry points** are listed that describe academic outcomes that are linked with grade-specific standards, described at successively lower levels of complexity at which specific students may be able to achieve them. **Access skills** are also listed that provide instructional strategies to expose students with the *most* significant cognitive disabilities to the concepts, content, and curriculum materials for each domain or conceptual category. Access skills are listed at the lowest grade level in each strand.

Figure 1
How to Determine the Appropriate Level of Complexity



Definition of Terms Used in the Resource Guide

The following terms are used to describe the information listed in the *Resource Guide to the 2011 Frameworks for Students with Disabilities—Mathematics:*

- **Access Skills** are developmental (communication or motor) skills that are addressed during instructional activities based on standards in the content area being assessed. *For example*, a student may participate in a Geometry lesson by maintaining a grasp of materials used in the lesson, such as 3D shapes, or by activating a pre-programmed voice output device to comment or make a request during a mathematics activity. Examples of access skills are listed at the lowest grade level in each mathematics domain or conceptual category in the Mathematics Resource Guide. Incorporating access skills into standard-based activities provides opportunities for students with the most significant disabilities to:
    - practice targeted skills in a variety of settings using a range of instructional approaches
    - be exposed to materials and concepts based on the general education curriculum
    - prepare *some* students with significant disabilities to address entry points based on grade-level standards in the future
- **Clusters** are groups of related standards (e.g., "Define, evaluate, and compare functions" in the Functions domain)
- **Cluster headings** describe the overall knowledge and skills addressed by the standards in the cluster (i.e., the "essence")
- **Conceptual Categories** are large groups of standards in a related area of Mathematics (e.g., Statistics and Probability) for students in high school.
- **Domains** are large groups of standards in a related area of Mathematics (e.g., Counting and Cardinality) for students in preK through grade 8.
- **Entry Points** are academic outcomes at successively lower levels of complexity (i.e., below grade-level expectations) that are aligned with each standard or cluster of standards. They are intended for use by educators to instruct students with disabilities who are performing below the grade-level expectations for other students in that grade. Entry points are:
    - aligned with the grade-level standard(s) on which it is based
    - modified below grade-level expectations
    - listed on a continuum approaching grade-level complexity (i.e., less to more complex)
    - used to identify measurable outcomes for a student who is taking the MCAS-Alt**.** Educators are required to identify a challenging, observable, and measurable skill for alternate assessment in each domain, as described in the *Educator's Manual for MCAS-Alt*.
- **Standards** define what students should understand and be able to do in each grade. Each standard is listed precisely as it appears in the *2011 Massachusetts Curriculum Framework for Mathematics.* Each standard is identified by an alpha-numeric code (e.g., Standard K.OA.A.5 - "Fluently add and subtract within 5").

MCAS Alternate Assessment

Resource Guide to the 2011 Massachusetts Curriculum Frameworks

for Students with Disabilities

(Incorporating the Common Core State Standards)

MATHEMATICS

Fall 2014

Table 1
Mathematics Domains and Conceptual Categories to be Assessed by MCAS-Alt

Mathematics
Progression from Pre-K Through High School

Domain (grades 3-8) / Conceptual Category (high school)	Grade Level										
	PK	K	1	2	3	4	5	6	7	8	HS (Choice of three)
Counting and Cardinality											
Operations and Algebraic Thinking					★	★					
Number and Operations in Base Ten							★				
Number and Operations—Fractions						★	★				
The Number System								★			
Ratios and Proportional Relationships								★	★		
Expressions and Equations										★	
Functions											⌘
Measurement and Data					★						
Geometry									★	★	⌘
Statistics and Probability											⌘
Number and Quantity											⌘
Algebra											⌘

KEY
★ = assessed by MCAS-Alt
⌘ = Select 3 of the 5 Conceptual Categories

Counting and Cardinality

	Standards	Entry Points	Access Skills
Pre-K	Page 9	Pages 10 – 13	Pages 10 – 14
K	Page 15	Page 16	

CONTENT AREA Mathematics
DOMAIN Counting and Cardinality

Pre-Kindergarten

Cluster	Standards as written	
Know number names and the counting sequence.	**MA.PK.CC.A.1**	Listen to and say the names of numbers in meaningful contexts.
	MA.PK.CC.A.2	Recognize and name written numerals 0–10.
Count to tell the number of objects.	**MA.PK.CC.B.3**	Understand the relationships between numerals and quantities up to ten.
Compare numbers.	**MA.PK.CC.C.4**	Count many kinds of concrete objects and actions up to ten, using one-to-one correspondence, and accurately count as many as seven things in a scattered configuration.
	MA.PK.CC.C.5	Use comparative language, such as more/less than, equal to, to compare and describe collections of objects.

ENTRY POINTS and ACCESS SKILLS for Counting and Cardinality Standards in Pre-K

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Know number names and the counting sequence.	♦ Respond to materials as they are counted ♦ Attend visually, auditorially, or tactilely to objects as they are counted ♦ Shift focus from materials to speaker counting materials ♦ Grasp materials as they are counted ♦ Release materials as they are counted ♦ Give materials as they are counted ♦ Move objects as they are counted ♦ Orient objects as they are counted (e.g., turn flowerpots upright) ♦ Manipulate objects with two hands as they are counted ♦ Locate objects partially hidden or out of sight to add or subtract to a collection of objects to be counted ♦ Use one object to act on another as objects are counted (e.g., use a pointer to tap)	♦ Identify numerals ♦ Identify number words ♦ Count by ones to 3 ♦ Match spoken number names and written numerals (e.g., one/1, 2/two), up to 3 ♦ Answer yes/no questions about counting	♦ Count by ones to 5, without objects ♦ Match spoken number names and written numerals (e.g., one/1, 2/two), up to 5 ♦ Sing songs/recite rhymes that involve counting up to 5.	♦ Match spoken number names and written numerals (e.g., one/1, 2/two), up to 10 ♦ Count by ones up to 10 without objects ♦ Sing songs/recite rhymes that involve counting up to 10.

ACCESS SKILLS (continued) for Counting and Cardinality Standards in Pre-K

Less Complex ⟷ **More Complex**

	ACCESS SKILLS **The student will:**	**ENTRY POINTS** **The student will:**
Know number names and the counting sequence. (continued)	♦ Adjust plane to move objects in counting activities (e.g., tip plank so that materials can be named in counting sequence as they fall) ♦ Construct using materials that have been counted in sequence (e.g., tower of blocks) ♦ Turn device on/off to participate in counting sequence activity (e.g., activate preprogrammed voice-generating device to recite number names) ♦ Imitate action in counting sequence activity ♦ Initiate cause-and-effect response in counting sequence activity (e.g., use switch to activate a number-naming cause-and-effect computer program) ♦ Sustain counting sequence activity through response ♦ Gain attention in counting sequence activity ♦ Make a request in counting sequence activity (e.g., request a turn to move the marker on a board game) ♦ Choose from an array of two during a counting sequence activity (e.g., choose materials to be counted	

ENTRY POINTS and ACCESS SKILLS for Counting and Cardinality Standards in Pre-K

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Count to tell the number of objects.	♦ Choose beyond an array of two during a counting sequence activity (e.g., choose materials to be counted) ♦ Follow directions in counting sequence activities (e.g., follow direction to "put the pencils in the box" as the teacher counts) ♦ Sustain activity through response during a counting activity	♦ Place a given number of manipulatives/ objects up to 3 ♦ Use fingers to represent a number up to 3 ♦ Match number names and/or numerals to small quantities of objects up to 3 ♦ Choose from two numerals the numeral that represents the number of manipulatives	♦ Place given number of manipulatives/ objects up to 5 ♦ Use fingers to represent a number up to 5 ♦ Match number names and/or numerals to small quantities of objects up to 5	♦ Place a given number of manipulatives/ objects up to 10 ♦ Use fingers to represent a number up to 10 ♦ Match number names and/or numerals to small quantities of objects up to 10

ENTRY POINTS and ACCESS SKILLS for Counting and Cardinality Standards in Pre-K

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		**ENTRY POINTS**	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Compare numbers.	♦ Respond to materials being compared by quantity or size ♦ Attend visually, auditorially, or tactilely to materials being compared by quantity or size ♦ Track materials being compared by quantity or size ♦ Shift focus from materials being compared by quantity or size to speaker ♦ Grasp materials being compared by quantity or size ♦ Release materials being compared by quantity or size ♦ Give materials being compared by quantity or size ♦ Move materials being compared by quantity or size ♦ Orient materials being compared by quantity or size ♦ Manipulate materials being compared by quantity or size ♦ Locate objects partially hidden or out of sight to compare by quantity or size to another set	♦ Demonstrate 1:1 correspondence by counting up to 3 objects, starting from one ♦ Distinguish between same/different quantities of similar objects ♦ Organize objects in a group to represent a quantity up to 3 ♦ Match quantity to quantity name ♦ Select (by counting) which group of objects has 1, 2, or 3 items	♦ Demonstrate 1:1 correspondence by counting up to 5 objects, starting from one ♦ Select by counting which group of objects has 1–5 items ♦ Identify the correct amount of scattered objects without counting, up to 3	♦ Demonstrate 1:1 correspondence by counting up to 7 objects, starting from one ♦ Select by counting which group of objects has 1–7 items. ♦ Identify the correct amount of scattered objects without counting, up to 5

ACCESS SKILLS (continued) for Counting and Cardinality Standards in Pre-K

Less Complex ⟷ **More Complex**

	ACCESS SKILLS **The student will:**	**ENTRY POINTS** **The student will:**
Compare numbers. (continued)	♦ Construct two objects using materials from two sets of materials being compared by quantity or size (e.g., build two block towers, with one set of 3 blocks and one with 5 blocks) ♦ Turn on voice-generating device to participate in an activity to compare materials by quantity or size ♦ Imitate action with materials being used to compare quantity or size ♦ Sustain activity comparing objects by size or quantity through response ♦ Gain attention in activity comparing objects by size and quantity (e.g., raise hand during comparison lesson on white board) ♦ Make a request in an activity comparing materials by size or quantity Choose from (or beyond) an array of two in an activity to compare materials by quantity	

CONTENT AREA Mathematics
DOMAIN Counting and Cardinality

Kindergarten

Cluster	Standards as written	
Know number names and the count sequence.	**K.CC.A.1**	Count to 100 by ones and by tens.
	K.CC.A.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
	K.CC.A.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).
Count to tell the number of objects.	**K.CC.B.4**	Understand the relationship between numbers and quantities; connect counting to cardinality.
	K.CC.B.4a	When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
	K.CC.B.4b	Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
	K.CC.B.4c	Understand that each successive number name refers to a quantity that is one larger.
	K.CC.B.5	Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
Compare numbers.	**K.CC.C.6**	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
	K.CC.C.7	Compare two numbers between 1 and 10 presented as written numerals.

ENTRY POINTS for
Counting and Cardinality Standards in Kindergarten

Less Complex ⟵ ⟶ **More Complex**

	The student will:	**The student will:**	**The student will:**
Know number names and the count sequence.	♦ Count to 20 by ones ♦ Starting with any number greater than one, count forward by ones to 5 ♦ Write numbers from 1-5 ♦ Match spoken number names and written numerals (e.g., one/1, 2/two), up to 15 ♦ Demonstrate 1:1 correspondence between objects by counting up to 15 objects, starting from one *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Count to 50 by ones ♦ Starting with any number greater than one, count forward by ones to 10 ♦ Write numbers from 1-10 ♦ Match spoken number names and written numerals (e.g., one/1, 2/two), up to 20 ♦ Demonstrate 1:1 correspondence between objects by counting up to 20 objects, starting from one	♦ Count to 50 by tens ♦ Starting with any number greater than one, count forward by ones to 20 ♦ Write numbers from 1-15 ♦ Demonstrate 1:1 correspondence between objects by counting up to 50 objects, starting from one *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Count to tell the number of objects.	♦ Count to answer "how many?" (line, array, circle) using various arrangements of up to 3 objects ♦ Select objects from a larger group, up to 3	♦ Count to answer "how many?" (line, array, circle) using various arrangements of up to 5 objects	♦ Count to answer "how many?" using various arrangements (line, array, circle) of up to 10 objects
Compare numbers.			♦ Compare two written numerals between 1 and 5

Operations and Algebraic Thinking

	Standards	Entry Points	Access Skills
Pre-K	Page 18	Page 19	Pages 19 – 20
K	Page 21	Pages 22 – 23	
1	Page 24	Pages 25 – 27	
2	Page 28	Pages 29 – 31	
3	Page 32	Pages 33 – 35	
4	Page 36	Pages 37 – 38	
5	Page 39	Page 40	

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Pre-Kindergarten

Cluster	Standards as written	
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	**MA.PK.OA.A.1**	Use concrete objects to model real-world addition (putting together) and subtraction (taking away) problems up through five.

ENTRY POINTS and ACCESS SKILLS for Operations and Algebraic Thinking Standards in Pre-K

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	♦ Respond to materials to be added, subtracted, and/or counted ♦ Attend visually, auditorially, or tactilely to materials to be added, subtracted, and/or counted ♦ Track object as it is added, subtracted, and/or counted from set ♦ Shift focus on materials as they are added, subtracted, and/or counted ♦ Grasp materials to be added, subtracted, and/or counted ♦ Release materials to be added, subtracted, and/or counted ♦ Give materials to be added, subtracted, and/or counted ♦ Move materials to be added, subtracted, and/or counted ♦ Orient objects/ materials added, subtracted, and/or counted (e.g., turn flowerpots upright as they are added, subtracted, and/or counted)	♦ Combine and take apart sets of objects to represent the concept of addition and subtraction ♦ Add ("put together") and subtract ("take away") using up to 2 manipulatives or objects ♦ Answer yes/no questions about "adding to" or "taking from"	♦ Create sets of "more," starting with 2 or 3 objects ♦ Create sets of "less" starting with 3 objects ♦ Compare groupings of various objects, using comparative terms up to 4 ♦ Show "one more" starting with 2 or 3 objects use manipulatives ♦ Show "one less" starting with 2 or 3 objects use manipulatives ♦ Illustrate concept of more or less	♦ Demonstrate "add to" using 5 or more objects or manipulatives ♦ Demonstrate "subtract from" using 5 or more manipulatives or objects

ACCESS SKILLS (continued) for Operations and Algebraic Thinking Standards in Pre-K

Less Complex ⟷ **More Complex**

	ACCESS SKILLS **The student will:**	**ENTRY POINTS** **The student will:**
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from (continued)	♦ Locate objects partially hidden or out of sight, to add to a set ♦ Use one object to act on another in an adding, subtracting, and/or counting activity (e.g., use a net to add or subtract blocks to sets) ♦ Construct/deconstruct or assemble/disassemble object by adding or subtracting pieces (e.g., add or subtract blocks from tower) ♦ Turn device on/off during an addition, subtraction, and/or counting activity (e.g., turn on voice-generating device to indicate add) ♦ Imitate action to add, subtract, and/or count ♦ Initiate cause-and-effect response (e.g., use switch to activate an adding, subtracting, and/or counting cause-and-effect program) ♦ Sustain addition, subtraction, and/or counting activity through response (e.g., when reading about adding, subtracting, and/or counting stops, vocalizes to request more) ♦ Gain attention in an addition, subtraction, and/or counting activity ♦ Make a request during adding, subtracting, and/or counting activity (e.g., to ask for help or request a turn to add or subtract materials) ♦ Choose from an array of two in an addition, subtraction, and/or counting lesson	

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Kindergarten

Cluster	Standards as written	
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	**K.OA.A.1**	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), and acting out situations, verbal explanations, expressions, or equations.
	K.OA.A.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
	K.OA.A.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
	K.OA.A.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
	K.OA.A.5	Fluently add and subtract within 5.

ENTRY POINTS for
Operations and Algebraic Thinking Standards in Kindergarten

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	♦ Add ("put together") 1, 2, and 3 fingers, manipulatives, or objects ♦ Subtract ("take away") 1, 2, and 3 fingers, manipulatives, or objects ♦ "Add one" or "take away one" using objects or pictures from sets of up to 3 ♦ Represent addition and subtraction word problems with up to 3 objects or manipulatives ♦ Decompose a number in more than one way using objects or manipulatives (e.g., 3 = 1 + 2; 3 = 2 + 1) ♦ Find the number that makes 3 when using the numbers 1 or 2 added to the given number (using fingers or sounds, e.g., claps) ♦ Find the number that makes 5 when added to a given number using fingers or manipulatives ♦ Count up to 5 objects to answer questions about how many altogether, using manipulatives ♦ Add ("put together") and subtract ("take away") within 3 using objects and/or manipulatives *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Add ("put together") and subtract ("take away") using the numbers 1–5 (using sounds, e.g., claps) ♦ Represent addition and subtraction using the numbers 1–5 (e.g., using manipulatives, acting out situations) ♦ "Add two" or "take away two" from sets of up to 5 using objects or pictures ♦ Represent addition and subtraction word problems with up to 5 objects/manipulatives, or with drawings ♦ Decompose numbers up to 5 in more than one way using drawings or objects (e.g., 5=2+3, 5=4+1) ♦ Find the number that makes 5 when added to a given number 1–4, (using manipulatives, objects, or drawings) ♦ Find the number that makes 5 when added to the given number and record the answer using a drawing and/or manipulatives ♦ Add ("put together") and subtract ("take away") within 5 using objects and manipulatives ♦ Count up to 10 objects to answer questions about how many altogether using manipulatives	♦ Select the symbol (+) to represent addition and the symbol (-) to represent subtraction upon request ♦ "Add two" or "take away two" from sets of up to 10 using objects or pictures ♦ Solve addition and subtraction problems by manipulating objects ♦ Express the meanings of "add" (make more) and "subtract" (make less) ♦ Represent addition and subtraction word problems with up to 10 objects/manipulatives, or with drawings ♦ Match numerals with up to 10 objects ♦ Decompose numbers less than or equal to 10 in more than one way and express as an equation (e.g., 7 = 2 + 5 and 7 = 4 + 3) ♦ Demonstrate two ways to decompose a set of objects to create the inverse (e.g., 1 + 2 = 3 to 3 – 1 = 2) ♦ Find the number that makes 10 when added to a given number (e.g., using objects)

ENTRY POINTS for
Operations and Algebraic Thinking Standards in Kindergarten

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. (continued)			*Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Grade 1

Cluster	Standards as written	
Represent and solve problems involving addition and subtraction.	**1.OA.A.1**	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
	1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
Understand and apply properties of operations and the relationship between addition and subtraction.	**1.OA.B.3**	Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)
	1.OA.B.4	Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.
Add and subtract within 20.	**1.OA.C.5**	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
	1.OA.C.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use mental strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
Work with addition and subtraction equations.	**1.OA.D.7**	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
	1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □.
	1.OA.D.9	Write and solve number sentences from problem situations that express relationships involving addition and subtraction within 20.

ENTRY POINTS for
Operations and Algebraic Thinking Standards in Grade 1

Less Complex ← → **More Complex**

	The student will:	The student will:	The student will:
Represent and solve problems involving addition and subtraction.	♦ Add and subtract up to three one-digit numbers using manipulatives or objects ♦ Identify sets containing more, less/fewer ♦ Show "add to", using manipulatives and examples up to 7 ♦ Show "take from", using manipulatives and examples up to 7 ♦ Show "put together," with manipulatives and examples up to 7 ♦ Show "take apart," with manipulatives and examples up to 7 ♦ Show "compare" with manipulatives and examples up to 7 ♦ Express "putting together' and "taking apart" with the appropriate language to describe any aspect of addition (plus, combined, added, more) ♦ Express "taking apart" with the appropriate language to describe any aspect of subtraction (minus, take away, less, remove, difference) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Solve problems involving addition and subtraction of one-digit numbers with a result no greater than 10 ♦ Solve word problems using addition and subtraction within 10 to "add to" involving unknowns in all positions. (e.g., using objects and drawing to represent the problem) ♦ Solve word problems using addition and subtraction within 10 to "take from" involving unknowns in all positions. (e.g., using objects and drawing to represent the problem) ♦ Solve word problems using addition and subtraction within 10 to "put together /take apart" involving unknowns in all positions. (e.g., using objects and drawing to represent the problem) ♦ Solve word problems using addition and subtraction within 10 to "compare" involving unknowns in all positions. (e.g., using objects and drawing to represent the problem)	♦ Add and subtract up to three one-digit numbers that total no more than 10 ♦ Use conventional mathematical signs and symbols to represent mathematical relationships in a word problem; i.e., (+), (-), and (=) ♦ Solve word problems using addition and subtraction within 15 to "add to" involving unknowns in all positions with equations and graphic organizers ♦ Solve word problems using addition and subtraction within 15 to "take from" involving unknowns in all positions with equations and graphic organizers ♦ Solve word problems using addition and subtraction within 15 to "put together/take apart" involving unknowns in all positions with equations and graphic organizers ♦ Solve word problems using addition and subtraction within 15 to "compare" involving unknowns in all positions with equations and graphic organizers *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 1

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand and apply properties of operations and the relationship between addition and subtraction	♦ Combine objects to demonstrate addition ("plus," "combined with," "more," "join") ♦ Take groups of objects apart to demonstrate subtraction ("minus," "take away," "less," "remove," "difference") ♦ Group objects to demonstrate the commutative property of addition (e.g., one object plus two objects is the same as two objects plus one object) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Show the commutative property of addition use pairs of numbers 1–5 (e.g., if 1 + 4 = 5 is known, then 4 + 1 = 5 is also known) ♦ Group objects to demonstrate the associative property of addition	♦ Show the associative property of addition use up to three one-digit numbers 1–10 where the first two digits are added together (e.g., 5 + 3 +2 = 5 + 5 = 10) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Add and subtract within 20.	♦ Create sets using pictures or objects ♦ Match numerals with up to 5 objects ♦ Indicate the number that results when adding one more to a given number, use manipulatives or visual representations or a number line (e.g., to find 1 + 4, count 4 objects, then one more) ♦ Subtract 1 from numbers up to 5 using concept of "one less" *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Add objects by grouping into combinations up to 5 (using manipulatives, fingers, etc.) ♦ Add and subtract numbers within 5 by using the relationship between addition and subtraction (e.g., if 2 + 3 = 5, then we know that 5 – 2 = 3) ♦ Match numeral with up to 10 objects	♦ Add and subtract one-digit numbers in number sentences ♦ Solve problems involving addition and subtraction using the numbers 1–10 ♦ Add and subtract within 10 using strategies such as counting on and using the relationship between addition and subtraction (e.g., knowing that 8 + 2 = 10, we know that 10 – 8 = 2) ♦ Match numeral with up to 15 objects *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for
Operations and Algebraic Thinking in Grade 1

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Work with addition and subtraction equations.	♦ Identify the terms and corresponding symbols for addition (+) and subtraction (-) ♦ Show the concept of trading equal amounts ♦ Add one object to each set express result ♦ Match one-to-one equivalents (e.g., put one object in each of multiple sets, match pairs of objects) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Solve simple addition and subtraction number sentences, using pictures, objects, and/or manipulatives ♦ Make comparisons between sets ♦ Determine missing objects or quantities within sets ♦ Create sets to demonstrate the meaning of "equal to" ♦ Create sets that have the same number of objects	♦ Identify the missing component in a number sentence, using symbols or objects (2 + ? = 4) ♦ Show equalities in number sentences (e.g., 3 + 1 = 2 + 2) ♦ Represent the relationships between groupings of objects using symbol of equality ♦ Express appropriate terms and/or corresponding symbol for "equal to" ♦ Represent mathematical relationships using conventional mathematical signs and symbols ♦ Solve number sentences that express relationships involving addition and subtraction within 10 *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Grade 2

Cluster	Standards as written	
Represent and solve problems involving addition and subtraction.	**2.OA.A.1**	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
Add and subtract within 20.	**2.OA.B.2**	Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two one-digit numbers.
	MA.2.OA.B.2a	By the end of grade 2, know from memory related subtraction facts of sums of two one-digit numbers.
Work with equal groups of objects to gain foundations for multiplication.	**2.OA.C.3**	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
	2.OA.C.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

ENTRY POINTS for
Operations and Algebraic Thinking Standards in Grade 2

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Represent and solve problems involving addition and subtraction.	♦ Identify the terms and corresponding symbols for addition (+) and subtraction (-) ♦ Add and subtract numbers within 20 using manipulatives ♦ Represent addition and subtraction word problems within 20 using manipulatives ♦ Manipulate number sentences in a given problem to show that the same sum can be achieved by adding numbers in different arrangements up to 20 (e.g., 1 + 2 = 3 and 2 + 1 = 3) ♦ Represent simple addition and subtraction number sentences with manipulatives ♦ Create number sentences up to 20 by manipulating objects ♦ Solve word one-step problems using addition and subtraction within 20 to "add to" involving unknowns in all positions with manipulatives ♦ Solve word problems using addition and subtraction within 20 to "put together/take apart" involving unknowns in all positions with manipulatives ♦ Solve one –step word problems using addition and subtraction within 20 to "take from" involving unknowns in all positions with manipulatives	♦ Identify terms and symbols for +, -, = ♦ Add and subtract numbers within 50 using manipulatives and drawings ♦ Represent one-step addition and subtraction word problems within 50 using manipulatives and drawings ♦ Express the inverse relationship between addition and subtraction within 50 using number sentences ♦ Identify common phrases used to indicate the need for addition and subtraction ♦ Create number sentences with unknown numbers up to 50 using pictures/objects ♦ Solve word one-step problems using addition and subtraction within 50 to "add to" involving unknowns in all positions with manipulatives and drawings ♦ Solve word problems using addition and subtraction within 50 to "put together/take apart" involving unknowns in all positions with manipulatives and drawings ♦ Solve one-step word problems using addition and subtraction within 50 to "take from" involving unknowns in all positions with manipulatives and drawings	♦ Solve one-step word problems using addition and subtraction within 100 to "add to" involving unknowns in all positions with equations and graphic organizers ♦ Solve one-step word problems using addition and subtraction within 100 to "take from" involving unknowns in all positions with equations and graphic organizers ♦ Solve one–step word problems using addition and subtraction within 100 to "put together/take apart" involving unknowns in all positions with equations and graphic organizers ♦ Solve one-step word problems using addition and subtraction within 100 to "compare" involving unknowns in all positions with equations and graphic organizers *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 3

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Represent and solve problems involving addition and subtraction. (continued)	♦ Solve one-step word problems using addition and subtraction within 20 to "compare" involving unknowns in all positions with manipulatives *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Solve one-step word problems using addition and subtraction within 50 to "compare" involving unknowns in all positions with manipulatives	♦
Add and subtract within 20.	♦ Demonstrate addition within 5 as "putting together" and subtraction as "taking away" using manipulatives and drawings *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Demonstrate addition within 10 as "putting together" and subtraction as "taking away" using manipulatives and drawings	♦ Demonstrate addition within 20 as "putting together" and subtraction as "taking away" using manipulatives and drawings *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Work with equal groups of objects to gain foundations for multiplica-tion.	♦ Create two equal groups from an even number of objects ♦ Create a 2xN array by arranging an even number of objects into groups of twos (N can be 1-10) ♦ Skip count objects by rows arranged in a 2xN array to demonstrate repeated addition (e.g., A carton of eggs with 2 rows of 6 eggs= 2 + 2 + 2 ♦ Group objects by multiples of two *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*		♦ Create sets to demonstrate the meaning of "equal to" ♦ Group and count by 2's up to 20 ♦ Group and count by 5's up to 20 ♦ Group and count by 10's up to 20 ♦ Determine if a quantity is odd or even by pairing two groups of up to 20 objects, using manipulatives and drawings ♦ Match two equal groupings with odd or even quantities with appropriate numeral to represent actual quantities up to 10 ♦ Use manipulatives or models to add the number of objects in an array with up to 5 rows and 5 columns

ENTRY POINTS for
Operations and Algebraic Thinking Standards in Grade 3

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Work with equal groups of objects to gain foundations for multiplication. (continued)	♦	♦	♦ Equally distribute an even number of objects up to 20 into 2–4 groups *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Grade 3

Cluster	Standards as written	
Represent and solve problems involving multiplication and division.	**3.OA.A.1**	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
	3.OA.A.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
	3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
	3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*
Understand properties of multiplication and the relationship between multiplication and division.	**3.OA.B.5**	Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.)* *3 × 5 × 2 can be found by 3 × 5 = 15 then 15 × 2 = 30, or by 5 × 2 = 10 then 3 × 10 = 30. (Associative property of multiplication.)* *Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
	3.OA.B.6	Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
Multiply and divide within 100.	**3.OA.C.7**	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of grade 3, know from memory all products of two one-digit numbers.
Solve problems involving the four operations, and identify and explain patterns in arithmetic.	**3.OA.D.8**	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.
	3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

ENTRY POINTS for
Operations and Algebraic Thinking Standards in Grade 3

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Represent and solve problems involving multiplication and division.	♦ Skip count the number of objects in an array to demonstrate repeated addition ♦ Represent and solve problems involving repeated addition ♦ Group objects together by 2s using manipulatives or drawings *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Represent repeated addition within 20 using two equal groups of objects (e.g., 2 groups of 3 is the same as 3 + 3) ♦ Group up to 10 objects in an array that demonstrates the concept of repeated addition ♦ Identify the number sentence that demonstrates the concept of multiplication using a given array of objects ♦ Solve simple multiplication number sentences with unknown numbers, using pictures, objects, and/or manipulatives	♦ Represent repeated addition within 30 using two equal groups of objects (e.g., 2 groups of 6 is the same as 6 + 6) ♦ Group up to 20 objects in an array that demonstrates the concept of repeated addition ♦ Create a number sentence showing multiplication using arrays of objects and manipulatives ♦ Identify common phrases used to indicate the need for multiplication such as "product", multiplication and "groups" ♦ Solve multiplication problems involving unknown variables in simple one-digit number sentences *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Understand properties of multiplication and the relationship between multiplication and division.	♦ Create groups of objects to demonstrate the commutative property of multiplication (e.g., 4 groups of 2 squares equals 2 groups of 4 squares) ♦ Illustrate the concept of multiplication using groups of objects ♦ Share up to 10 objects equally between 2 people (divide objects into 2 equal groups) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Match written problems that demonstrate the commutative properties of multiplication (e.g., 2 x 3 = 6 and 3 x 2 = 6) to visual representation ♦ Demonstrate the relationship between multiplication and division using manipulatives (e.g., 8 ÷ 2 is 8 objects divided into 2 groups of 4, which is equal to 4 groups of 2) ♦ Determine the unknown quantity in a multiplication equation (within 10) using manipulatives and drawings (e.g., how many groups of 4 objects is equal to 4?)	♦ Demonstrate the commutative property of multiplication and addition using arrays (e.g., 2 groups of 3 objects is equal to 3 groups of 2 objects) ♦ Create number sentences to demonstrate the relationship between multiplication and division (e.g., 8 ÷ 2 = 4 is the same as 4 X 2 = 8) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Operations and Algebraic Thinking Standards in Grade 3

Less Complex ← → **More Complex**

Understand properties of multiplication and the relationship between multiplication and division. (continued)		♦ Illustrate the concept of division using groups of objects ♦ Visually represent problem situations by sharing up to 20 objects equally between 2 and 5 people	♦ Determine the unknown quantity in a multiplication equation (within 20) using manipulatives and drawings (e.g., how many groups of 5 objects is equal to 15?) ♦ Identify common phrases used to indicate the need for multiplication and division ♦ Write a number sentence representing up to 30 objects divided into equal groups of 2, 5, or 10 ♦ Demonstrate the commutative property of multiplication using manipulatives ♦ Illustrate division by making equal-sized groups using models (e.g., 2 equal-sized groups up to 10) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Multiply and divide within 100.	♦ Solve multiplication problems with a multiplier 1, 2, 5, or 10 ♦ Solve division problems within 100 with a divisor of 1, 2, 5, or 10 *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Solve multiplication problems with a multiplier 1, 2, 3, 4, 5, or 10 ♦ Solve division problems within 100 with a divisor of 1, 2, 3, 4, 5, or 10	♦ Solve multiplication problems with multipliers of 1–10 ♦ Solve division problems within 100 with divisors of 1–10 *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS to Standards for Operations and Algebraic Thinking in Grade 3

Less Complex ← → **More Complex**

Solve problems involving the four operations, and identify and explain patterns in arithmetic.	♦ Solve one-step word problems using addition or subtraction ♦ Identify the required operation to solve one-step word problem ♦ Reproduce a given number pattern ♦ Reproduce a given shape pattern ♦ Extend simple repeating shape patterns, given a model or example ♦ Extend simple repeating number patterns, given a model or example ♦ Use rounding strategies to make estimates ♦ Predict what will come next in a shape pattern with three shapes (e.g., square, circle, triangle) ♦ Extend a simple addition or subtraction pattern (e.g., adding by 2s, subtracting by 3s) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Identify the missing component in a multiplication number sentence within 20, using symbols or objects (e.g., identify the array that matches the sentence 2 x ? = 10) ♦ Solve one-step word problems using multiplication or division ♦ Identify a number or shape pattern ♦ Create repeating shape patterns given a rule (e.g., given the rule ABCABC, student draws ∇◊□∇◊□)	♦ Choose whether an estimate or exact amount is needed in a given situation ♦ Identify the missing component in a multiplication number sentence within 30, using symbols or objects (e.g., identify the array that matches the sentence 5 x ? = 15) ♦ Solve two-step word problems using any of the four operations ♦ Use estimation to determine the reasonableness of a solution to a one-step word problem ♦ Represent a word problem using a number sentence and appropriate symbols (+, -, ×, ÷, =) ♦ Use estimation to approximate the solution to a one-step word problem ♦ Create a pattern based on a given rule ♦ Identify the rule of a given number or shape pattern ♦ Create a simple addition or subtraction pattern *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Grade 4

Cluster	Standards as written	
Use the four operations with whole numbers to solve problems.	**4.OA.A.1**	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 ☐7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
	4.0A.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
	4.OA.A.3	Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Gain familiarity with factors and multiples.	**4.OA.B.4**	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
Generate and analyze patterns.	**4.OA.C.5**	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

ENTRY POINTS for
Operations and Algebraic Thinking Standards in Grade 4

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Use the four operations with whole numbers to solve problems.	♦ Identify the missing number in an addition or subtraction problem using manipulatives or drawings ♦ Solve a one-step word problem involving addition, subtraction, multiplication, or division ♦ Visually illustrate a one-step addition or subtraction problem with an unknown number using manipulatives or drawings ♦ Determine the reasonableness of an answer to an addition or subtraction problem using estimation strategies of "more" and "less" ♦ Replace unknown numbers with given values in two digit by two digit addition/subtraction sentences *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Identify the missing number in number sentences involving addition and subtraction (e.g., 25 - ? = 21) ♦ Visually represent single-digit multiplication problems using strategies such as equal-sized groups, repeated addition, equal-sized jumps on a number line, or area models ♦ Solve number sentences that represent one-step multiplication and division word problems ♦ Solve number sentences derived from one-step addition or subtraction word problems using a letter or symbol for the unknown quantity ♦ Replace unknown numbers with given values in simple mathematical sentences (e.g., solve 2 × ? If ? = 2, 3, or 4) ♦ Determine the reasonableness of an answer to multiplication problems using estimation strategies of repeated addition	♦ Identify the missing number in number sentences involving multiplication and division (e.g., 10 ÷ ? = 5) ♦ Solve a two-step word problem ♦ Solve number sentences derived from one-step word problems using letters or symbols for the unknown quantity ♦ Determine the reasonableness of an answer to division problems using estimation strategies (e.g., successive subtraction) ♦ Visually represent division problems with single-digit divisors using successive subtraction, equal-sized jumps on a number line, sharing, or partitioning *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Gain familiarity with factors and multiples.	♦ Find multiples of 2 using manipulatives or drawings ♦ Find factors within 10 using manipulatives, tables, or drawings *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Find multiples of 2 and 3 using manipulatives or drawings ♦ Find factors within 20 using manipulatives, tables, or drawings	♦ Find multiples of 2, 3, and 5 using manipulatives, drawings, or symbols ♦ Find factors within 50 using manipulatives, tables, or drawings ♦ Identify prime numbers and numbers that can be factored *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for
Operations and Algebraic Thinking Standards in Grade 4

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Generate and analyze patterns.	♦ Reproduce a given number pattern ♦ Reproduce a given shape pattern ♦ Extend repeating shape patterns, given a model or example ♦ Extend simple repeating number patterns, given a model or example *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Identify a number or shape pattern ♦ Predict what will come next in a shape pattern with three or more shapes (e.g., square, circle, triangle) ♦ Extend a simple addition or subtraction pattern (e.g., adding by 2s, subtracting by 3s)	♦ Create a pattern based on a given rule ♦ Identify the rule of a given number or shape pattern ♦ Create a simple addition or subtraction pattern *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Operations and Algebraic Thinking

Grade 5

Cluster	Standards as written	
Write and interpret numerical expressions.	**5.OA.A.1**	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
	5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "Add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
Analyze patterns and relationships.	**5.OA.B.3**	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

ENTRY POINTS for
Operations and Algebraic Thinking Standards in Grade 5

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Write and interpret numerical expressions.	♦ Solve numerical expressions in a given problem using the appropriate operation(s) ♦ List the Order of Operations given a numerical expression involving addition and subtraction (e.g., 10 – 5 + 2 =?) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Use the Order of Operations to solve numerical expressions involving addition and subtraction ♦ Use the Order of Operations to create a numerical expression based on a real-world problem involving addition before subtraction ♦ Use the Order of Operations to solve two-step numerical expressions	♦ Use the Order of Operations to evaluate numerical expressions involving addition, subtraction, and multiplication ♦ Use the Order of Operations to create a numerical expression based on a real-world problem involving multiplication before addition ♦ Use the Order of Operations to solve numerical expressions containing grouping symbols (e.g., 3 x (2 - 1) = 3 x 1 = 3) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Analyze patterns and relationships.	♦ Extend a simple numerical pattern ♦ Count by twos to create a numerical pattern *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Identify a numerical pattern ♦ Extend a numerical pattern using a rule for addition and subtraction ♦ Count by twos and fives to create a numerical pattern ♦ Identify what is missing from a numerical pattern	♦ Explain the rule of a given numerical pattern ♦ Create a numerical pattern using a rule for addition, subtraction, and/or multiplication ♦ Count by twos, fives, tens, and hundreds to create a numerical pattern *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Number and Operations in Base Ten

	Standards	Entry Points	Access Skills
K	Page 42	Page 43	Pages 43 – 44
1	Page 45	Pages 46 – 47	
2	Page 48	Page 49 – 50	
3	Page 51	Page 52	
4	Page 53	Page 54	
5	Page 55	Pages 55 – 56	

CONTENT AREA Mathematics
DOMAIN Number and Operations in Base Ten

Kindergarten

Cluster	Standards as written	
Work with numbers 11–19 to gain foundations for place value.	**K.NBT.A.1**	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

ENTRY POINTS and ACCESS SKILLS for Number and Operations in Base Ten Standards in Kindergarten

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Work with numbers 11–19 to gain founda-tions for place value.	♦ Respond to materials as they are counted ♦ Attend visually, auditorially, or tactilely to materials as they are counted ♦ Shift focus from materials to speaker counting materials ♦ Grasp materials as they are counted ♦ Release materials as they are counted ♦ Give materials as they are counted ♦ Move objects as they are counted ♦ Orient objects as they are counted (e.g., turn flowerpots upright) ♦ Manipulate objects with two hands as they are counted ♦ Locate objects partially hidden, or out of sight, to add or subtract to a collection of objects to be counted ♦ Use one object to act on another as objects are counted (e.g., use a pointer to tap) ♦ Adjust plane to move objects in counting activities (e.g., tip plank so that materials can be named in counting sequence as they fall)	♦ Count by ones up to 10 ♦ Represent a number of objects (up to 5) with a written numeral (with 0 representing a count of no objects). ♦ Compose numbers from 1 to 9 to create 10 and record each composition by using objects. ♦ Decompose 10 into two numbers between 1 and 9 and record each decomposition by using objects ♦ Answer yes/no questions related to numbers, quantities or counting	♦ Compose numbers from 1 to 9 to create 10, record each composition by using objects and/or drawings. ♦ Decompose 10 into two numbers between 1 and 9; record each decomposition by using objects and/or drawings	♦ Compose numbers from 1 to 9 to create 10 and record each composition by using objects, drawings and/or equations (e.g., 1 + 9 = 10) ♦ Decompose 10 into two numbers between 1 and 9 and record each decomposition by using objects, drawings and/or equations (e.g., 10 - 1 = 9)

ACCESS SKILLS (continued) for Number and Operations in Base Ten Standards in Kindergarten

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	**ENTRY POINTS The student will:**
Work with numbers 11–19 to gain foundations for place value. (continued)	♦ Construct using materials that have been counted in sequence (e.g., tower of blocks) ♦ Turn device on/off to participate in counting sequence activity (e.g., activate preprogrammed voice-generating device to recite number names) ♦ Imitate action in counting sequence activity ♦ Initiate cause-and-effect response in counting sequence activity (e.g., use switch to activate a number-naming cause-and-effect computer program) ♦ Sustain counting sequence activity through response ♦ Gain attention in counting sequence activity ♦ Make a request in counting sequence activity (e.g., request a turn to move the marker on a board game) ♦ Choose from an array of two during a counting sequence activity (e.g., choose materials to be counted) ♦ Choose beyond an array of two during a counting sequence activity (e.g., choose materials to be counted) ♦ Follow directions in counting sequence activities (e.g., follow direction to "Put the pencils in the box" as the teacher counts)	

CONTENT AREA Mathematics
DOMAIN Number and Operations in Base Ten

Grade 1

Cluster	Standards as written	
Extend the counting sequence.	**1.NBT.A.1**	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
Understand place value.	**1.NBT.B.2**	Understand that the two digits of a two-digit number represent amounts of tens and ones.
	Understand the following (2a, 2b, and 2c) as special cases:	
	1.NBT.B.2a	10 can be thought of as a bundle of ten ones—called a "ten."
	1.NBT.B.2b	The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
	1.NBT.B.2c	The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
	1.NBT.B.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
Use place value understanding and properties of operations to add and subtract.	**1.NBT.C.4**	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
	1.NBT.C.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
	1.NBT.C.6	Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

ENTRY POINTS for
Number and Operations in Base Ten Standards in Grade 1

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Extend the counting sequence.	♦ Count by ones up to 20 *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Count by tens to 100. ♦ Represent a number of objects (between 10 and 20) with a numeral by writing and/or using layered place value cards or flip books ♦ Count forward beginning from a given number up to 20 within the known sequence (e.g., count on from 13)	♦ Count by ones up to 100 ♦ Represent a number of objects (up to 99) with a numeral by writing and/or using layered place value cards or flip books ♦ Count forward beginning from a given number up to 100 within the known sequence (e.g., count on from 23). *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Understand place value.	♦ Create groups/bundles of 10 from 10, 20, 30 single objects ♦ Visually represent quantities between 11 and 19 as a bundle of "ten" and the appropriate number of "ones" use manipulatives on a place value mat ♦ Label bundles with numerals of up to twenty objects grouped/bundled into tens and ones on a place value mat ♦ Compare objects bundled into one "ten" or up to 9 "ones" using the terms greater than, equal to, and less than *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Visually represent quantities up to 50 as bundles of 10 ones (as "tens"), and up to 9 ones (as "ones") using manipulatives on a place value mat ♦ Use numerals to accurately label groups of up to fifty objects grouped/bundled into tens and ones on a place value mat ♦ Compare up to 50 objects grouped into tens and ones using the terms "greater than," "equal to," and "less than"	♦ Use numerals to accurately label groups of objects (up to 99) grouped/bundled into tens and ones on a place value mat ♦ Visually represent quantities up to 99 as bundles of "tens" and the appropriate number of "ones" using manipulatives on a place value mat or drawings ♦ Compare objects grouped into tens and ones using symbols (<, >, =) up to 50 *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for
Number and Operations in Base Ten Standards in Grade 1

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Use place value understand-ing and properties of operations to add and subtract.	♦ Add within 20 based on place value strategies use a visual representation (manipulatives, place value mats, drawings, and/or technology) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Add within 20 using equations based on place value strategies and a visual representation (manipulatives, place value mats, drawings, and/or technology) ♦ Identify "ten more" than a given two-digit number (using manipulatives, drawings, sounds, fingers, or counting by ones)	♦ Add and subtract within 5 using equations and visual representations based on place value strategies to (manipulatives, place value mats, drawings and/or technology) ♦ Identify "ten more" or "ten less" than a given two-digit number (using manipulatives bundled into tens and ones, place value mats, drawings, and/or counting on fingers) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Number and Operations in Base Ten

Grade 2

Cluster	Standards as written	
Understand place value.	**2.NBT.A.1**	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
	Understand the following (1a and 1b) as special cases:	
	2.NBT.A.1a	100 can be thought of as a bundle of ten tens—called a "hundred."
	2.NBT.A.1b	The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
	2.NBT.A.2	Count within 1000; skip-count by 5s, 10s, and 100s.
	2.NBT.A.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
	2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
Use place value understanding and properties of operations to add and subtract.	**2.NBT.B.5**	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
	2.NBT.B.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.
	2.NBT.B.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
	2.NBT.B.8	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
	2.NBT.B.9	Explain why addition and subtraction strategies work, using place value and the properties of operations.

ENTRY POINTS for
Number and Operations in Base Ten Standards in Grade 2

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand place value.	♦ Skip-count place value units by 100 within 1000 ♦ Using place value strategies, demonstrate the concept of trading equal amounts (e.g., 10 singles = a bundle of 10; 10 bundles = a bundle of 100) ♦ Represent a number of objects (up to 200) bundled into hundreds, tens and ones with a numeral by writing, using layered place value cards and/or flip books ♦ Create groups/bundles of 100 from bundles of "tens" ♦ Compare objects grouped into one hundred or up to 9 tens and up to 9 ones using the terms greater than, equal to, and less than (e.g., 100 is greater than 8 tens and 5 ones) ♦ Show numbers in expanded form up to 99 (e.g., 11 can be shown as 10 + 1; 12 can be shown as 10 + 2) ♦ Use number names to accurately label groups of objects to 99 into tens and ones ♦ Add and subtract single-digit numbers *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Skip-count place value units by 10 within 100 ♦ Use numerals to accurately label groups of objects (up to 500) grouped/bundled into hundreds, tens and ones (i.e. by writing, using layered place value cards and/or flip books) ♦ Use number names to accurately label groups of objects (up to 500) grouped/bundled into hundreds, tens and ones ♦ Visually represent quantities up to 500 as bundles of 10 tens (as "hundreds"), 10 ones (as "tens"), and up to 9 ones (as "ones") using manipulatives and place value mats ♦ Express the "hundreds" digit, the "tens" digit and "ones" digit of a written numeral between 11 to 499 by using manipulatives, place value mats and/or technology ♦ Compare objects grouped into hundreds, tens and ones using the terms "greater than," "equal to," and "less than," up to 200 ♦ Show numbers in expanded form up to 200 (e.g., 111 can be shown as 100 + 10 + 1; 125 can be shown as 100 + 20 + 5) ♦ Add and subtract double - digit numbers	♦ Skip-count place value units by 5 within 100 ♦ Use numerals to accurately label groups of objects (up to 999) grouped/bundled into hundreds, tens and ones (i.e. by writing, using layered place value cards and/or flip books) ♦ Use number names to accurately label groups of objects (up to 999) grouped/bundled into hundreds, tens and ones ♦ Visually represent quantities up to 999 as bundles using manipulatives and place value mats ♦ Compare objects grouped into hundreds, tens and ones using symbols (=, >, <) up to 500 ♦ Add and subtract three-digit numbers *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for
Number and Operations in Base Ten Standards in Grade 2

Less Complex ← → **More Complex**

Use place value understanding and properties of operations to add and subtract.	♦ Solve column addition and subtraction problems of 2 two-digit numbers that are multiples of ten using place value strategies (including manipulatives and drawings) ♦ Demonstrate the relationship between addition and subtraction within 10 using equations and place value materials (e.g., 5 + 3 = 8; 8 - 3 = 5) ♦ Add and subtract 100 to or from a given set of up to 900 objects *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Solve column addition and subtraction problems of two-digit numbers use place value strategies including manipulatives and drawings ♦ Add ten and subtract 10 to a given set of up to 100 objects ♦ Add up to three two-digit numbers using place value manipulatives	♦ Solve column addition and subtraction problems of three-digit numbers use place value strategies including manipulatives and drawings ♦ Demonstrate the relationship between addition and subtraction within 20 using number sentences and place value materials (e.g., 5 + 12 = 17; 17 - 5 = 12) ♦ Add and subtract ten to or from a given set of up to 900 objects using strategies based on place value ♦ Add up to three two-digit numbers using strategies based on place value and/or place value manipulatives *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Number and Operations in Base Ten

Grade 3

Cluster	Standards as written	
Use place value understanding and properties of operations to perform multi-digit arithmetic.	**3.NBT.A.1**	Use place value understanding to round whole numbers to the nearest 10 or 100.
	3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
	3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 • 80, 5 • 60) using strategies based on place value and properties of operations.

ENTRY POINTS for
Number and Operations in Base Ten Standards in Grade 3

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Use place value understanding and properties of operations to perform multi-digit arithmetic.	♦ Round whole three-digit numbers to the nearest 100 using place value materials *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Round whole two-digit numbers to the nearest 10 using place value materials	♦ Multiply one-digit numbers by a multiple of 10 (in the range of 10-90) using manipulatives, repeated addition, skip counting by tens or place value strategies ♦ Round whole three-digit numbers to the nearest 10 using place value materials *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Number and Operations in Base Ten

Grade 4

Cluster	Standards as written	
Generalize place value understanding for multi-digit whole numbers.	**4.NBT.A.1**	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
	4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
	4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.
Use place value understanding and properties of operations to perform multi-digit arithmetic.	**4.NBT.B.4**	Fluently add and subtract multi-digit whole numbers using the standard algorithm.
	4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
	MA.4.NBT.B.5a	Know multiplication facts and related division facts through 12 × 12.
	4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

ENTRY POINTS for
Number and Operations in Base Ten Standards in Grade 4

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Generalize place value understanding for multi-digit whole numbers.	♦ Compose and decompose multi-digit numbers by their place values using expanded form and base-ten materials such as layered place value cards or flip books ♦ Represent a three-digit whole number, to demonstrate that the digit in tens place represents ten times what it represents in the ones place and the digit in the hundreds place represents ten times what it represents in the tens place using place value manipulatives (e.g., in number 324, the 3 represents 30 bundles of tens and the 2 represents 20 units of ones) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Compare two numbers within 100 using "more than," "less than," or "equal to" (e.g., using place value materials) ♦ Interpret multi-digit whole numbers using base-ten materials, layered place value cards and/or flip books "express" answer ♦ Represent a four-digit whole number, demonstrate using place value manipulatives, (e.g., in number 5,324, the 5 represents 50 hundreds, the 3 represents 30 tens and the 2 represents 20 ones)	♦ Express multi-digit whole numbers using numerals, place value materials, layered place value cards, or flip books ♦ Show numbers in expanded form (e.g., 1,111 can be shown as 1,000 + 100 + 10 + 1; 6,125 can be shown as 6,000 + 100 + 20 + 5) ♦ Compare numbers within 1,000 using the symbols >, <, and = using numerals and place value materials, layered place value cards and/or flip books ♦ Round whole numbers to the nearest 10, 100 using place value *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Use place value understanding and properties of operations to perform multi-digit arithmetic.	♦ Multiply a one-digit number by at least a two-digit number using place-value manipulatives ♦ Multiply one-digit numbers by a multiple of 100 and/or 1,000 using manipulatives, place value strategies, or the properties of multiplication ♦ Divide up to a two-digit number by a one-digit number without remainders using place value materials, rectangular arrays and/or area models *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Multiply a one-digit number by at least a two-digit number using an equation , array or area model ♦ Divide up to a three-digit number by a one-digit number without remainders using equations and place value materials, rectangular arrays, and/or area models ♦ Solve division problems up to two digits by one digit, using the relationship between multiplication and division to demonstrate that, for example, 76÷4=19 because 19x4=76	♦ Multiply a two-digit number by a two digit-number using equations and an array or area model ♦ Divide up to a three-digit number by a one-digit number with remainders using equations and place value materials, rectangular arrays and/or area models ♦ Solve three-digit by one digit division problems with a quotient that is a multiple of ten, using reasoning about place value (e.g., 420 ÷ 7 = 60 because 42 ÷ 7 is 6 and 42 ten divided by 7 is 6 tens) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Number and Operations in Base Ten

Grade 5

Cluster	Standards as written	
Understand the place value system.	**5.NBT.A.1**	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
	5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
	5.NBT.A.3	Read, write, and compare decimals to thousandths.
	5.NBT.A.3a	Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
	5.NBT.A.3b	Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
	5.NBT.A.4	Use place value understanding to round decimals to any place.
Perform operations with multi-digit whole numbers and with decimals to hundredths.	**5.NBT.B.5**	Fluently multiply multi-digit whole numbers using the standard algorithm.
	5.NBT.B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
	5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

ENTRY POINTS for
Number and Operations in Base Ten Standards in Grade 5

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand the place value system.	♦ Express decimals to the hundredths using place value materials (e.g., flip books) ♦ Compare decimals to the tenths using symbols (=, >, <) (e.g., flip books and/or manipulatives) ♦ Connect money to decimals by rounding to the dollar (e.g.$2.57 becomes $3) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Express decimals to the thousandths using place value materials (e.g., flip books) ♦ Compare decimals to the hundredths using symbols (=, >, <) and place value materials (e.g., flip books and/or manipulatives) ♦ Express decimals to the hundredths using expanded form ♦ Show that in a three-digit whole number, a digit in one place represents 1/10 what it represents in the place to the left using place value manipulatives (e.g., use place value materials to show that the 5 in 356 is 1/10 the value of the 5 in 514) ♦ Connect money to decimals by rounding to the nearest dime (e.g., $2.57 becomes $2.60)	♦ Write decimals to the hundredths using place value materials (e.g., flip books) ♦ Write decimals to the hundredths using expanded form ♦ Rounds decimals to the nearest tenth ♦ Use whole number exponents to denote powers of ten (e.g., show that 10 x 10 x 10 = 1,000 and 10^3 *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for
Number and Operations in Base Ten Standards in Grade 5

Less Complex ← → **More Complex**

Perform operations with multi-digit whole numbers and with decimals to hundredths.	*See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Divide up to a three-digit number by a two-digit number without remainders using equations and place value materials, rectangular arrays or area models ♦ Add decimals to tenths using equations and place value materials, concrete models or drawings or strategies based on properties of operations ♦ Subtract decimals to tenths, using equations and place value materials, concrete models, drawings or strategies based on properties of operations ♦ Multiply a whole number by a decimal in the tenths using estimation (e.g., 6 x 0.19 is close to 6 x $0.20 which is $1.20 my number will be one whole and a decimal.) ♦ Multiply a whole number by a decimal in the tenths using number lines ♦ Multiply a whole number by a decimal in the tenths using repeated addition ♦ Multiply a whole number by a decimal in the tenths using coins ♦ Multiply a whole number by a decimal in the tenths using concrete models, drawings or strategies based on properties of operations (e.g., using a hundreds grid, I can shade 19 squares 6 times to show on whole grid and 14 hundredths or 1.14)	♦ Divide up to a three-digit number by a two-digit number with remainders using equations and place value materials, rectangular arrays or area models ♦ Add decimals to hundredths equations use place value materials, concrete models, drawings or strategies based on properties of operations ♦ Subtract decimals to hundredths equations use place value materials, concrete models, drawings or strategies based on properties of operations ♦ Divide a number that includes a decimal to tenths by a whole number using estimation based on properties of operations (e.g., 7.6÷4 is close to 8÷4, my answer will be close to 2) ♦ Divide a number that includes a decimal to tenths by a whole number using repeated subtraction based on properties of operations ♦ Divide a number that includes a decimal to tenths by a whole number using coins based on properties of operations (e.g., share $7.60 with 4 people; each person will get 1 dollar and 9 dimes. My answer is 1.90) ♦ Divide a number that includes a decimal to tenths by a whole number using concrete models, drawings and/or strategies based on properties of operations *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Number and Operations – Fractions

	Standards	Entry Points	Access Skills
3	Page 59	Page 60	Pages 60 – 61
4	Pages 62 – 63	Page 64 – 65	
5	Pages 66 – 67	Page 68	

CONTENT AREA Mathematics
DOMAIN Number and Operations–Fractions

Grade 3

Cluster	Standards as written	
Develop understanding of fractions as numbers.	**3.NF.A.1**	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.
	3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram.
	3.NF.A.2a	Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
	3.NF.A.2b	Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
	3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
	3.NF.A.3a	Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
	3.NF.A.3b	Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
	3.NF.A.3c	Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.
	3.NF.A.3d	Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

ENTRY POINTS and ACCESS SKILLS for Number and Operations–Fractions Standards in Grade 3

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Develop understanding of fractions as numbers.	♦ Respond to materials that demonstrate objects that can be divided into equal parts ♦ Attend visually, auditorially, or tactilely to materials that demonstrate objects that can be divided into equal parts ♦ Track materials that demonstrate that objects can be divided into equal parts ♦ Shift focus from materials that demonstrate that objects can be divided into equal parts ♦ Grasp materials that demonstrate that objects can be divided into equal parts ♦ Use two hands to hold materials that demonstrate that objects can be divided into equal parts ♦ Release materials that demonstrate that objects can be divided into equal parts ♦ Move materials that demonstrate that objects can be divided into equal parts	♦ Identify concepts of whole and half using manipulatives and/or familiar objects ♦ Partition a whole into 2, 3 or 4 equal parts using visual models or manipulatives ♦ Compare fractions of the same whole to determine which is larger ♦ Manipulate equal parts of an object to create a whole ♦ Represent understanding of whole and half using visual models ♦ Answer yes/no questions about fractions ♦ Match a visual representation of a fraction to a fractional number line ♦ Create equivalent fractions using manipulatives	♦ Identify parts of a whole using concrete objects (e.g., 1/2, 1/4) ♦ Compare fractions using the terms "greater than," "less than," or "equal to" ♦ Match visual representation of simple fractions to the name of the fraction ♦ Determine whether two fractions are equivalent by using a fraction number line, manipulatives and/or technology) ♦ Manipulate whole objects to make two, three, or four parts of a whole ♦ Compare parts of a whole (quarters, thirds, halves) to determine relative size of each (1/2, 1/3, 1/4) using manipulatives or visual models ♦ Locate unit fractions* on a number line ♦ Partition the number line from zero to 1 into 2, 4 or 8 equal parts (e.g., folding a paper in to equal parts)	♦ Identify parts of a whole using visual fraction models and/or objects (e.g., 1/2, 1/3, 1/4, 1/6, 1/8) ♦ Record results of the comparisons of two fractions using symbols (<, =, >) ♦ Create visual representation of simple fractions ♦ Order simple fractions on a number line ♦ Label simple fractions on a number line ♦ Determine the size of a unit fraction* by using same sized pieces to create a whole. (e.g., If you need 4 pieces to make a whole the unit fraction* is ¼) *Continue to address skills and concepts that approach grade-level expectations in this cluster* * **Unit Fraction**: a fraction with a numerator of one

ACCESS SKILLS (continued) for Number and Operations–Fractions Standards in Grade 3

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	**ENTRY POINTS The student will:**
Develop understanding of fractions as numbers. (continued)	♦ Orient materials that demonstrate that objects can be divided into equal parts ♦ Locate objects partially hidden or out of sight (e.g., remove barrier to expose part that when added to object equals the whole object) ♦ Turn device on/off to participate in an activity on fractions (e.g., turn on voice-generating device) to comment on fraction activity ♦ Imitate action required to divide object ♦ Initiate cause-and-effect response (e.g., turn on technology tool) to activate fraction activity ♦ Sustain activity through response in a fraction based activity ♦ Gain attention (e.g., request a turn) with fraction materials ♦ Make a request in a fraction based activity ♦ Choose from an array of two in a fraction based activity (e.g., choose materials to be divided into equal parts)	

CONTENT AREA Mathematics
DOMAIN Number and Operations–Fractions

Grade 4

Cluster	Standards as written	
Extend understanding of fraction equivalence and ordering.	**4.NF.A.1**	Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
	4.NF.A.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	**4.NF.B.3**	Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
	4.NF.B.3a	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
	4.NF.B.3b	Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8; 21/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.
	4.NF.B.3c	Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
	4.NF.B.3d	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
	4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
	4.NF.B.4a	Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
	4.NF.B.4b	Understand a multiple of ***a***/***b*** as a multiple of **1**/***b***, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (**2**/**5**) as 6 × (**1**/**5**), recognizing this product as **6**/**5**.* (In general, *n* × (***a***/***b***) = (*n* × *a*)/*b*.)
	4.NF.B.4c	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions.	**4.NF.C.5**	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.
	4.NF.C.6	Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100 ; describe a length as 0.62 meters; locate 0.62 on a number line diagram.
	4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

ENTRY POINTS for
Number and Operations–Fractions Standards in Grade 4

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Extend understanding of fraction equivalence and ordering.	♦ Demonstrate fractions equivalent to ½ using manipulatives and/or technology (e.g., 2/4 = ½) ♦ Determine which of two fractions with like denominators represents a larger part of a whole (e.g., compare 2 fourths and 3 fourths) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Show equivalent fractions using manipulatives and/or technology (e.g., 2/6 = 1/3) ♦ Determine which of two fractions represents a larger part of a whole (fraction manipulatives or visual models)	♦ Generate multiple pairs of equivalent fractions using manipulatives and/or technology ♦ Compare two fractions, visually demonstrate which is greater and less than the benchmark of a half using manipulative or technology ♦ Demonstrate, using manipulatives, that the whole is equal to the sum of the partitioned parts (e.g., 4/4 = 1) ♦ Compare visual models of fractions using symbols (<,>,=) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	♦ Add simple fractions using manipulatives, visual models and/or technology (e.g., showing that two halves equal a whole or two fourths equal a half) ♦ Subtract simple fractions using visual models (e.g., manipulatives and/or technology) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Add unit fractions* with the same denominator with more than two addends, using manipulatives or technology, (e.g., showing that ¼ + ¼ + ¼ = ¾ or ½ + ½ + ½=3/2) ♦ Multiply fractions using visual models and repeated addition (e.g., showing that ¼ x 3=¾ and ¼ + ¼ + ¼ = ¾) ♦ Add and subtract fractions, using manipulatives, visual models and/or technology * **Unit Fraction**: a fraction with a numerator of one	♦ Add unit fractions with the same denominator with more than one addend using both visual models and equations (e.g., showing that ¼ + ¼ + ¼ = ¾ or ½ + ½ + ½ =3/2) ♦ Subtract fractions with the same denominator from a mixed number ♦ Add and subtract fractions with the same denominator using mixed numbers ♦ Solve addition and subtraction fraction word problems with the same denominator use manipulatives ♦ Represent a mixed number as an equivalent fraction using visual models or technology, given the written form (e.g., show that 1½ =3 halves)

ENTRY POINTS for
Number and Operations–Fractions Standards in Grade 4

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (continued)			♦ Show the related equation when multiplying fractions, using repeated addition and/or visual models ♦ Multiply a whole number times a non-unit fraction (e.g., 3 X 2/4 =2/4 + 2/4 + 2/4 also equals 3 groups of 2/4 and each 2/4 = ¼ + ¼) using manipulatives, visual models and/or technology ♦ Solve a multiplication word problem involving fractions using manipulatives *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Understand decimal notation for fractions, and compare decimal fractions.	♦ Locate a decimal on a number line *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Show that a fraction with a denominator of ten is equivalent to a fraction with a denominator of 100 by using manipulative, visual models and/or technology	♦ Express a fraction with a denominator of ten as an equivalent fraction with a denominator of 100 ♦ Use decimal notation for fractions with denominators of ten. (e.g., 2/10=0.2) ♦ Compare two decimals to the tenths by reasoning about their size using symbols (=,<,>) or visual model (e.g., number line) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Number and Operations–Fractions

Grade 5

Cluster	Standards as written	
Use equivalent fractions as a strategy to add and subtract fractions.	**5.NF.A.1**	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)
	5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2 .
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	**5.NF.B.3**	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size ¾. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
	5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
	5.NF.B.4a	Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q × b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd .)
	5.NF.B.4b	Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
	5.NF.B.5a	Interpret multiplication as scaling (resizing), by comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
	5.NF.B.5b	Interpret multiplication as scaling (resizing), by explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1.

Apply and extend previous understand-ings of multiplication and division to multiply and divide fractions. (continued)	**5.NF.B.6**	Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
	5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
	5.NF.B.7a	Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12, because (1/12) × 4 = 1/3.
	5.NF.B.7b	Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20, because 20 × (1/5) = 4.
	5.NF.B.7c	Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

ENTRY POINTS for
Number and Operations–Fractions Standards in Grade 5

Less Complex ⟵ ⟶ **More Complex**

	The student will:	**The student will:**	**The student will:**
Use equivalent fractions as a strategy to add and subtract fractions.	*See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Add fractions with unlike denominators, by using manipulatives or technology to create equivalent fractions with like denominators. ♦ Subtract fractions with unlike denominators, by using manipulatives or technology to create equivalent fractions with like denominators	♦ Add fractions with unlike denominators, creating sums greater than one by using manipulatives or technology to create equivalent fractions with like denominators ♦ Subtract fractions from a mixed number with unlike denominators by using manipulatives or technology to create equivalent fractions with like denominators ♦ Add and subtract fractions with unlike denominators using mixed numbers ♦ Solve word problems with fractions with unlike denominators using manipulatives or technology *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	♦ Solve real-world problems involving division of a whole into equal parts (e.g., divide a candy bar to share with four friends) ♦ Solve real-world problems using addition of fractions ♦ Solve real-world problems using subtraction of fractions *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials* * **Unit Fraction**: a fraction with a numerator of one	♦ Solve real-world problems using multiplication of fractions by whole numbers ♦ Represent connection between fractions and division with the use of visual models, manipulatives or technology (e.g., show 8/4 can be represented as 8 candy bars into 4 groups results in each group getting 2 candy bars) ♦ Solve real-world problems involving division a whole into equal parts (e.g., divide 3/4 of a candy bar among three friends) ♦ Multiply a whole number by a fraction greater than 1 use visual representations to correspond to the product. (e.g., show 4 x 3/2 = 12/2 by using ½ pieces)	♦ Solve real-world problems using multiplication of fractions by fractions using manipulatives, visual models and/or technology ♦ Multiply a whole number by a fraction less than 1 use visual representation to correspond to the product. (e.g., show 4 x ½ = 2 by using ½ pieces) ♦ Solve a real-world problems by dividing a whole number by a unit fraction* (e.g., 3 pizzas are divided into fourths there are 12 pieces) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

The Number System

	Standards	Entry Points	Access Skills
5	Page 70	Page 71	Pages 71 – 72
6	Pages 73 – 74	Pages 74 – 77	
7	Page 78	Page 79	
8	Page 80	Page 81	

Grade 5

Cluster	Standards as written	
Gain familiarity with concepts of positive and negative integers.	**MA.5.NS.A.1**	Use positive and negative integers to describe quantities such as temperature above/below zero, elevation above/below sea level, or credit/debit.

ENTRY POINTS and ACCESS SKILLS for The Number System Standards in Grade 5

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Gain familiarity with concepts of positive and negative integers.	♦ Respond to materials as they are counted ♦ Attend visually, auditorially, or tactilely to materials as they are counted ♦ Shift focus from materials to speaker counting materials ♦ Grasp materials as they are counted ♦ Release materials as they are counted ♦ Give materials as they are counted ♦ Move objects as they are counted ♦ Orient objects as they are counted (e.g., turn flowerpots upright) ♦ Manipulate objects with two hands as they are counted ♦ Locate objects partially hidden or out of sight to add or subtract to a collection of objects to be counted ♦ Use one object to act on another as objects are counted (e.g., use a pointer to tap) ♦ Adjust plane to move objects in counting activities (e.g., tip plank so that materials can be named in counting sequence as they fall)	♦ Answer yes/no questions about zero	♦ Describe real-world quantities as greater than or less than zero	♦ Represent the concept of positive and negative (less than zero) given real-world situations (e.g., money, temperature, sea level, wins/losses)

ACCESS SKILLS (continued) for The Number System Standards in Grade 5

Less Complex ← → **More Complex**

	ACCESS SKILLS The student will:	**ENTRY POINTS The student will:**
Gain familiarity with concepts of positive and negative integers. (continued)	♦ Construct using materials that have been counted in sequence (e.g., tower of blocks) ♦ Turn device on/off to participate in counting sequence activity (e.g., activate preprogrammed voice-generating device to recite number names) ♦ Imitate action in counting sequence activity ♦ Initiate cause-and-effect response in counting sequence activity (e.g., use switch to activate a number-naming cause-and-effect computer program) ♦ Sustain counting sequence activity through response ♦ Gain attention in counting sequence activity ♦ Make a request in counting sequence activity (e.g., request a turn to move the marker on a board game) ♦ Choose from an array of two during a counting sequence activity (e.g., choose materials to be counted) ♦ Choose beyond an array of two during a counting sequence activity (e.g., choose materials to be counted) ♦ Follow directions in counting sequence activities (e.g., follow direction to "Put the pencils in the box" as the teacher counts) ♦ Respond to materials as they are counted in sequence ♦ In the context of an academic activity on addition (putting together and adding to) and subtraction (taking apart and taking from), respond to materials to be added or subtracted ♦ Attend visually, auditorially, or tactilely to materials to be added or subtracted ♦ Track object as it is added or subtracted from set	

Grade 6

Cluster	Standards as written	
Apply and extend previous under-standings of multiplication and division to divide fractions by fractions.	**6.NS.A.1**	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3.* (In general, (*a*/*b*) ÷ (*c*/*d*) = *ad*/*bc*.) *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
Compute fluently with multi-digit numbers and find common factors and multiples.	**6.NS.B.2**	Fluently divide multi-digit numbers using the standard algorithm.
	6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
	6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the Distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4*(*9 + 2*)*.*
	MA.6.NS.B.4a	Apply number theory concepts, including prime factorization and relatively prime numbers, to the solution of problems.
Apply and extend previous under-standings of numbers to the system of rational numbers.	**6.NS.C.5**	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
	6.NS.C.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
	6.NS.C.6a	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
	6.NS.C.6b	Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
	6.NS.C.6c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
	6.NS.C.7	Understand ordering and absolute value of rational numbers.

Apply and extend previous understandings of numbers to the system of rational numbers. (continued)	**6.NS.C.7a**	Interpret statements of inequality as statements about the relative positions of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
	6.NS.C.7b	Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*
	6.NS.C.7c	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*
	6.NS.C.7d	Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*
	6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

ENTRY POINTS for
The Number System Standards in Grade 6

Less Complex ⟵ ⟶ **More Complex**

	The student will:	**The student will:**	**The student will:**
Apply and extend previous understand-ings of multiplication and division to divide fractions by fractions.	♦ Demonstrate, using manipulatives, that the whole is equal to the sum of the partitioned parts (e.g., 4/4 =1/4+1/4+1/4+1/4= 1) ♦ Identify parts of a whole using concrete objects (e.g., 1/2, 1/4) ♦ Match visual representations of simple fractions to the name of the fraction ♦ Add and subtract simple fractions from a whole, using manipulatives or technology ♦ Convert an improper fraction to a mixed number using manipulatives ♦ Convert an improper fraction to a mixed number ♦ Use manipulatives and strategies to show repeated division (e.g., paper folding) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Order simple fractions with the same denominator ♦ Identify parts of a whole using visual fraction models (e.g., 1/2, 1/3, 1/4, 1/6, 1/8) ♦ Create visual representations of simple fractions ♦ Add simple fractions, using manipulatives or technology, (e.g., showing that ½ + ½ = 2/2 and ¼ + ¼ =2/4) ♦ Subtract simple fractions from a whole, using manipulatives or technology, (e.g., showing that 2/2 - 1/2 = 1/2 and 4/4 – 1/4 = 3/4) ♦ Add and subtract fractions with the same denominator using mixed numbers	♦ Order simple fractions with same numerator ♦ Create visual representations of addition and subtraction of simple fractions ♦ Subtract fractions with the same denominator from a mixed number ♦ Solve word problems involving fractions and mixed numbers ♦ Convert an improper fraction to a mixed number using numerals *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Compute fluently with multi-digit numbers and find common factors and multiples.	♦ Add and subtract whole numbers ♦ Multiply and divide whole numbers ♦ Multiply two-digit numbers by one-digit numbers using manipulatives and drawings ♦ Divide two-digit numbers by one-digit numbers using manipulatives and drawings ♦ Solve repeated addition problems where the addends are the same (i.e., 5 + 5 + 5 = 15 is equal to three groups of five) using concrete manipulatives and/or a calculator ♦ Identify numbers that can be divided by 2	♦ Order numbers using tenths (e.g., 0.1) ♦ Represent tenths (0.1 or 1 out of 10) using manipulatives or drawings ♦ Multiply three-digit numbers by one-digit numbers ♦ Divide two-digit numbers by one-digit numbers (no remainders) ♦ Add numbers including decimals to tenths ♦ Subtract numbers including decimals to tenths ♦ Multiply numbers including decimals to tenths	♦ Order numbers using hundredths (e.g., 0.01) ♦ Represent hundredths (0.01 or 1 out of 100) using manipulatives or drawings ♦ Multiply up to three-digit numbers by two-digit numbers ♦ Divide three-digit numbers by one-digit numbers (no remainders) ♦ Add numbers including decimals to hundredths ♦ Subtract numbers including decimals to hundredths ♦ Multiply numbers including decimals to hundredths

ENTRY POINTS for
The Number System Standards in Grade 6

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Compute fluently with multi-digit numbers and find common factors and multiples. (continued)	♦ Divide groups of manipulatives into groups of equal amounts ♦ Demonstrate the variety of number combinations that when added together equal a whole number up to 10, using manipulatives (e.g., 10 = 1 + 9 or 2 + 8 or 3 + 7 or 4 + 6 or 5 + 5) ♦ Identify equivalent forms of common decimals/fractions less than one whole (.5 = 1/2) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Divide numbers, including decimals, to tenths ♦ Find the product of whole numbers to 20 via multiple algorithms (e.g., different ways to get to 20 = 10 x 2; 2 x 10)	♦ Divide numbers, including decimals, to hundredths ♦ Find the factors of numbers to 40 (e.g., 1, 2, 4, 5, 8, 10, 20, 40) ♦ Identify prime numbers up to 40 *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Apply and extend previous understand-ings of numbers to the system of rational numbers.	♦ Construct a horizontal number line demonstrating positive numbers and negative numbers and zero ♦ Label a number line to order positive numbers ♦ Represent real-world quantities using a vertical number line (e.g., temperature: below zero, zero, above zero, etc.) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Construct a number line to demonstrate the distance from zero for positive and negative numbers ♦ Determine how many units separate two points on a number line that includes positive and negative numbers ♦ Show the distance from zero (absolute value) for positive and negative numbers (e.g., |-7| = 7 because -7 is 7 units from zero) ♦ Use a number line to order integers and common fractions less than one whole ♦ Compare distances from zero using a number line (e.g., |-7| > |3| because -7 is farther away from zero and |-5| = |5| =5 because they are both 5 units from zero) ♦ Order integers and common fractions using a number line to less than one whole	♦ Identify the negative numbers on a number line that are equal distance from zero as its positive pair ♦ Plot points in all four quadrants on a coordinate grid ♦ Add and subtract the absolute value of rational numbers ♦ Solve problems using a number line to demonstrate the process *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for
The Number System Standards in Grade 6

Less Complex ← → **More Complex**

	The student will:	The student will:	The student will:
Apply and extend previous understandings of numbers to the system of rational numbers. (continued)		♦ Label a number line to order integers as positive (+) and negative (-) ♦ Plot points in the first quadrant on a coordinate grid	

CONTENT AREA Mathematics
DOMAIN The Number System

Grade 7

Cluster	Standards as written	
Apply and extend previous understand-ings of operations with fractions to add, subtract, multiply, and divide rational numbers.	**7.NS.A.1**	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
	7.NS.A.1a	Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
	7.NS.A.1b	Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
	7.NS.A.1c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
	7.NS.A.1d	Apply properties of operations as strategies to add and subtract rational numbers.
	7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
	7.NS.A.2a	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the Distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
	7.NS.A.2b	Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
	7.NS.A.2c	Apply properties of operations as strategies to multiply and divide rational numbers.
	7.NS.A.2d	Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
	7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

ENTRY POINTS for
The Number System Standards in Grade 7

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Apply and extend previous understand-ings of operations with fractions to add, subtract, multiply, and divide rational numbers.	♦ Add positive and negative numbers using a number line, manipulatives and/or zero sums (e.g., 5 + (-3),-3 + 2, 2 + (-7)) ♦ Identify positive and negative numbers on a number line *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Add positive and negative numbers in real-world situations ♦ Subtract positive and negative numbers using number lines, manipulatives, zero sums, etc. (e.g., 6 - 2, 6 - 8, 5- (-1), -5 - 3, -5 - (-2)) ♦ Use multiplication and division to solve real-world problems	♦ Add and subtract positive and negative numbers ♦ Divide a fraction by another fraction with different denominators using various approaches (divide numerators, divide denominators, invert and multiply) ♦ Apply the rules for multiplying signed numbers (e.g., -7 x 4 = -28; -5 x -4 = 20) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN The Number System

Grade 8

Cluster	Standards as written	
Know that there are numbers that are not rational, and approximate them by rational numbers.	**8.NS.A.1**	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
	8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$ *show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

ENTRY POINTS for
The Number System Standards in Grade 8

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Know that there are numbers that are not rational, and approximate them by rational numbers.	♦ Order decimals on a number line ♦ Compare decimals (e.g., greater than, less than, equal to) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Convert fractions to decimal equivalent ♦ Estimate square roots of rational numbers ♦ Determine if a number is rational by deciding if it is (or can be written as) a fraction or a repeating or terminating decimal (e.g., 2.33333…, 3.567, $\sqrt{25}$, $\sqrt[3]{8}$, but not $\sqrt{10}$)	♦ Plot rational and irrational numbers on a number line ♦ Solve problems involving square roots (e.g., the area of a square is 20, estimate the length of a side) *Continue to address skills and concepts that approach grade-level expectations in this cluster* **Note:** An *irrational number* is one that cannot be expressed as a quotient of two integers, e.g. $\sqrt{2}$. A number is irrational if and only if it cannot be written as a repeating or terminating decimal.

Ratios and Proportional Relationships

	Standards	Entry Points	Access Skills
6	Page 83	Pages 84	Pages 84 – 85
7	Page 86	Page 87	

CONTENT AREA Mathematics
DOMAIN Ratios and Proportional Relationships

Grade 6

Cluster	Standards as written	
Understand ratio concepts and use ratio reasoning to solve problems.	**6.RP.A.1**	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
	6.RP.A.2	Understand the concept of a unit rate ***a*/*b*** associated with a ratio *a*:*b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* [2]
	6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
	6.RP.A.3a	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
	6.RP.A.3b	Solve unit rate problems, including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then, at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
	6.RP.A.3c	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
	6.RP.A.3d	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
	MA.6.RP.A.3e	Solve problems that relate the mass of an object to its volume.

[2] Expectations for unit rates in this grade are limited to non-complex fractions.

ENTRY POINTS and ACCESS SKILLS for Ratios and Proportional Relationships Standards in Grade 6

Less Complex ← → **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Understand ratio concepts and use ratio reasoning to solve problems.	♦ Respond to materials that demonstrate ratios and proportional relationships ♦ Attend visually, auditorially, or tactilely to materials that demonstrate ratios and proportional relationships ♦ Track materials that demonstrate ratios and proportional relationships ♦ Shift focus from materials that demonstrate ratios and proportional relationships ♦ Grasp materials that demonstrate ratios and proportional relationships ♦ Use two hands to hold materials that demonstrate ratios and proportional relationships ♦ Release materials used to demonstrate ratios and proportional relationships ♦ Move materials used to demonstrate ratios and proportional relationships ♦ Orient materials used to demonstrate ratios and proportional relationships	♦ Describe or express situations in which two quantities have a relationship ♦ Create ratios among objects sorted by attribute ♦ Create a model that represents the relationship of two given quantities, using drawings, manipulatives, or technology ♦ Identify the number of objects in a real-life problem situation using ratios (e.g., boys to girls) ♦ Compare fractions with same numerators or same denominators and record the comparisons using the symbols <, >, or =. ♦ Compare decimals using the symbols <, >, or =. ♦ Express percent equivalents using shapes, manipulatives, or drawings (e.g., ½ or a circle = 50 %) ♦ Express the mathematical relationship of two quantities as a ratio (e.g. the ratio of beaks to wings is 1:2) ♦	♦ Express ratios in simplest form using part-to-part given in real-life problems (e.g., 3 blue marbles to 9 green marbles is a ratio of 1:3) ♦ Express ratios in simplest form (e.g., 3 blue marbles to 9 green marbles is a ratio of 1:3) ♦ Compare fractions with different numerators or different denominators and record the comparisons using the symbols <, >, or =. ♦ Identify a unit rate (e.g., 5 for $1.00 or 5:1) ♦ Express fraction to percent equivalents using 100s chart or drawings (e.g., 50/100 = 50 %) ♦ Name the ordered pair that describes the location of a given point. graphed in the first quadrant on a coordinate plane	♦ Express a ratio from a real-life example in three different ways (e.g., 3 to 4, 3:4, 3/4) ♦ Express ratios in simplest form using part-to-whole given in real-life problems (e.g., blue marbles to the whole group containing blue, red, and green marbles) ♦ Compare two fractions by comparing each to a benchmark fraction (e.g., 2/5 < 1/2 and 5/8 > 1/2 so 2/5 < 5/8 ♦ Compare given unit rates and decide which is the" better buy" (i.e., the number of units of one quantity per unit of the other quantity) ♦ Plot equivalent ratios in the first quadrant of a coordinate plane ♦ Express fraction to decimal to percent as equivalents ♦ Calculate unit rates in real-life problems to make comparisons ♦ Plot points in all 4 quadrants of a coordinate plane

ACCESS SKILLS (continued) for Ratios and Proportional Relationships Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS **The student will:**	**ENTRY POINTS** **The student will:**
Understand ratio concepts and use ratio reasoning to solve problems. (continued)	♦ Locate objects partially hidden or out of sight (e.g., remove barrier) to expose a ratio ♦ Use one object to act on another used to demonstrate ratios ♦ Turn on/off technology used to demonstrate ratios and proportional relationships (e.g., turn on voice-generating device to describe a relationship using "to/for every" language) ♦ Imitate action to create proportional relationships ♦ Initiate cause-and-effect response (e.g., turn on technology tool to activate ratio computer program) ♦ Sustain ratio and proportional relationship activity through response ♦ Gain attention during a ratio activity) ♦ Make a request during ratio activity ♦ Choose from an array of two in adding and/or subtracting activity (e.g., choose materials to be distributed in a ratio and proportional relationship activity)	

CONTENT AREA Mathematics
DOMAIN Ratios and Proportional Relationships

Grade 7

Cluster	Standards as written	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	**7.RP.A.1**	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*
	7.RP.A.2	Recognize and represent proportional relationships between quantities.
	7.RP.A.2a	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table, or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
	7.RP.A.2b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
	7.RP.A.2c	Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
	7.RP.A.2d	Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.
	7.RP.A.3	Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

ENTRY POINTS for
Ratios and Proportional Relationships Standards in Grade 7

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Analyze proportional relationships and use them to solve real-world and mathematical problems.	♦ Identify the factor used to obtain equivalent ratios using manipulatives ♦ Describe the context of a ratio relationship, using "rate language", (e.g., this recipe has 4 cups of flour to 2 cups of sugar, so there are 2 cups of flour for each cup of sugar.) ♦ Find equivalent ratios to a given ratio in a real-life problem, using manipulatives ♦ Graph pairs of values displayed in ratio tables on a coordinate grid in first quadrant only ♦ Determine the percent of a quantity (e.g., what number is 50% of 8?) ♦ Write equivalent percents and fractions to demonstrate percents as "out of 100" ♦ Identify expressions that are equivalent (e.g., 6 + 5 = 3 + 8; or 2 x 8 = 4 x 4) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Identify equivalent ratios to fractions ♦ Fill in missing values in tables of equivalent ratios ♦ Graph pairs of values displayed in ratio tables on a coordinate grid in all four quadrants ♦ Solve single-step percent problems using proportional relationships (e.g., find unknown amount, %, or base) ♦ Calculate the base in a percent problem (e.g., 4 is 50% of what number?) ♦ Compute unit rates associated with ratios of whole numbers, including ratios of lengths and other quantities measured in like units (e.g., if 5 apples cost $10, what does one apple cost?) ♦ Solve one-step equations of the form *6x=12*	♦ Generate equivalent ratios to fractions ♦ Solve proportions where one missing quantity is represented by a variable (e.g., 3/6 = X/18) ♦ Solve for a unit rate in a real-life problem (e.g., if Sam runs 6 miles in ½ hour, what is his rate in mph?) ♦ Explain why a fraction a/b is equivalent to a fraction (n X a) / (n X b) by using visual fraction models. ♦ Create tables of equivalent ratios ♦ Solve equations that represent proportional relationships in real-life (e.g., use *d = rt* to find time if *d =* 120, *r =* 60, solve for t) ♦ Solve percent problems (e.g., 4 is what percent of 8?) ♦ Determine if given points, graphed on a coordinate grid lie on a straight line that passes through the origin *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Expressions and Equations

	Standards	Entry Points	Access Skills
6	Page 89 – 90	Pages 91 – 95, 97	Pages 91 – 98
7	Page 99	Pages 100 – 101	
8	Page 102 – 103	Pages 104 – 105	

CONTENT AREA Mathematics
DOMAIN Expressions and Equations

Grade 6

Cluster	Standards as written	
Apply and extend previous understandings of arithmetic to algebraic expressions.	**6.EE.A.1**	Write and evaluate numerical expressions involving whole-number exponents.
	6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers.
	6.EE.A.2a	Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
	6.EE.A.2b	Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
	6.EE.A.2c	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = ½$.*
	6.EE.A.3	Apply the properties of operations to generate equivalent expressions. *For example, apply the Distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the Distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
	6.EE.A.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
Reason about and solve one-variable equations and inequalities.	**6.EE.B.5**	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
	6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
	6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.
	6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.	**6.EE.B.9**	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Apply and extend previous understand-ings of arithmetic to algebraic expressions.	♦ Respond to materials to add, subtract, and/or count known and unknown quantities ♦ Attend visually, auditorially, or tactilely to materials to add, subtract, and/or count known and unknown quantities ♦ Track materials used to add, subtract, and/or count known and unknown quantities and create graphs ♦ Shift focus from materials used to add, subtract, and/or count known and unknown quantities, to speaker ♦ Grasp materials used to add, subtract, and/or count known and unknown quantities ♦ Use two hands to hold materials used to add, subtract, and/or count known and unknown quantities ♦ Release materials used to add, subtract, and/or count known and unknown quantities ♦ Move materials used to add, subtract, and/or count known and unknown quantities	♦ Create expressions involving addition and subtraction with numbers and with letters representing unknown numbers in real-life situations (e.g., "number of miles plus seven," can be written as "m + 7") ♦ Evaluate expressions involving addition and subtraction with numbers and with letters given the value of the unknown number (e.g., evaluate 6 – n when n = 2, n = 3, etc.) ♦ Represent repeated addition using equal groups of objects	♦ Rewrite numerical expressions with whole number exponents (e.g., 2 x 2 = 2^2; 5 to the third power = 5 x 5 x 5 = 5^3) ♦ Create expressions involving multiplication with numbers and with letters representing unknown numbers in real-life situations (e.g., "three times the number of marbles in a bag" can be written as "3n") ♦ Evaluate expressions involving multiplication with numbers and with letters representing unknown numbers given the value of the unknown number (e.g., when x = 4, find 6x) ♦ Identify when two numerical expressions are equivalent (e.g., 4 + 4 + 4 = 3 x 4)	♦ Create expressions involving any operation with numbers and with a letter representing unknown number in real-life situations (e.g., A class that has 32 students has 20 boys and n girls, 20 + n = 32) ♦ Evaluate expressions involving one or more operations with numbers and with letters representing unknown numbers given the value of the unknown number (e.g., evaluate the expression 3n - 5, when n = 8; student writes: 3 n 8 – 5=19) ♦ Create equivalent expressions of commutative property of addition (e.g., 5+3=3+5) ♦ Create equivalent expressions of commutative property of multiplication (e.g., 3x4=4x3) ♦ Create equivalent expressions involving the distributive property (e.g. 3(2 + 7) = (3 X 2) + (3 X 7))

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS	ENTRY POINTS		
	The student will:	**The student will:**	**The student will:**	**The student will:**
Apply and extend previous understand-ings of arithmetic to algebraic expressions. (continued)	♦ Orient materials used to add, subtract, and/or count known and unknown quantities (e.g., orient icon pictures used to label axis) ♦ Manipulate objects used to add, subtract, and/or count known and unknown quantities ♦ Locate objects partially hidden or out of sight needed to add, subtract, and/or count known and unknown quantities (e.g., remove barrier to expose materials) ♦ Use one object to act on another to add, subtract, and/or count known and unknown quantities (e.g., use scissors to cut materials) ♦ Turn on technology used to add, subtract, and/or count known and unknown quantities (e.g., turn on technology tool to add and subtract) ♦ Imitate action used to add, subtract, and/or count known and unknown quantities (e.g., imitate classmate attaching icon to add)	♦ Determine the equivalence of numerical expressions involving addition when one of the terms consists of two numbers (e.g., Does 6 + (2 + 8) = 6 + 10? ♦ Create a numerical expression involving addition and/or subtraction that represents a real-world problem ♦ Use Order of Operations to evaluate a numerical expression involving addition and subtraction from left to right ♦ Represent addition expressions using tables, charts, and drawings (e.g., 4 blocks plus 3 blocks is 4 + 3) ♦ Create groups of objects with equal amounts in multiple ways ♦ Represent repeated addition using equal groups of objects or numbers (e.g., 5+5+5 is the same as 5x3) ♦ Create an expression with multiplication from an array of objects or manipulatives	♦ Apply Order of Operations to produce equivalent expressions (e.g., 3 + 4 x 2 is equivalent to 3 + 8 ♦ Simplify numerical expressions involving addition and subtraction when there are multiple terms (e.g., (4+2) + (7-4) = 6+3=9) ♦ Represent multiplication expressions using tables, charts, drawings, manipulatives, or technology (e.g., 5 blocks in 3 baskets) ♦ Use Order of Operations to evaluate a numerical expression involving multiplication and division from left to right ♦ Evaluate expressions involving addition and subtraction by substituting the unknown quantity with more than one given value (e.g., find 4+n, if n=6, 7, or 8)	♦ Simplify numerical expressions involving the distributive property (e.g., 4(3 + 2) = (4 x 3) + (4 x 2) = 12 + 8 ♦ Evaluate expressions involving multiplication by substituting the unknown quantity with more than one given value (e.g., find 3r, if r = 2, 3, or 4) ♦ Use Order of Operations to evaluate a numerical expressions involving multiplication, division, addition and/or subtraction ♦ Use Order of Operations to evaluate a numerical expression that contains parentheses and up to four operations ♦ Create a numerical expression for multiplication or division given a real-world problem (e.g., 20 cookies shared by 5 people; student writes 20 ÷ 5) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

Less Complex ← → **More Complex**

	ACCESS SKILLS		**ENTRY POINTS**	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Apply and extend previous understand-ings of arithmetic to algebraic expressions. (continued)	♦ Initiate cause-and-effect response during an adding/subtracting/ counting activity (e.g., turn on technology tool to activate addition computer program) ♦ Sustain adding/subtracting/ counting activity through response (e.g., using preprogrammed voice-generating device comment) ♦ Gain attention during adding/subtracting/ counting activity (e.g., raise hand, vocalize) ♦ Make a request during an activity to add, subtract, and/or count known and unknown quantities (e.g., request a turn) ♦ Choose from an array of two in an adding, subtracting, and/or counting activity (e.g., choose materials to be used in adding activity)	♦ Demonstrate the commutative property of addition using manipulatives (e.g., 3 pencils + 2 pencils = 2 pencils + 3 pencils) ♦ Demonstrate the commutative property of multiplication using arrays		

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Reason about and solve one-variable equations and inequalities. (continued)	♦ Attend visually, auditorially, or tactilely to materials to add, subtract, and/or count known and unknown quantities ♦ Track materials used to add, subtract, and/or count known and unknown quantities ♦ Shift focus from materials used to add, subtract, and/or count known and unknown quantities, to speaker ♦ Grasp materials used to add, subtract, and/or count known and unknown quantities ♦ Use two hands to hold materials used to add, subtract, and/or count known and unknown quantities ♦ Release materials used to add, subtract, and/or count known and unknown quantities ♦ Move materials used to add, subtract, and/or count known and unknown quantities	♦ Match groups of objects to demonstrate the meaning of "equal to," "greater than," and "less than" ♦ Provide the range of solutions to inequalities when one of the number quantities is unknown (e.g., ? < 18, solution: anything less than 18) ♦ Answer yes/no questions about equations and inequalities ♦ Distinguish between additive and multiplicative comparison when presented with a real-world situation ♦ Interpret products of numbers (e.g., interpret 2 x 9 as 2 groups of 9 objects) ♦ Interpret sums of numbers (e.g., interpret 4 + 6 as 6 more than 4) ♦ Solve number sentences within 20 using addition or subtraction when one of the number quantities is unknown using tables, manipulatives, technology, or drawings	♦ Provide the range of solutions to inequalities within 99 involving addition or subtraction when one of the number quantities is unknown (e.g., 31 - ? < 18) ♦ Compare number quantities using the symbols <, =, > ♦ Write (express) inequalities involving addition or subtraction within 20 using the terms "greater than" and "less than" when one of the number quantities is unknown, using tables, manipulatives, technology, or drawings ♦ Assess the reasonableness of answers to equations using neutral computation ♦ Describe a context in which a product is expressed (e.g., describe 2 x 7 as 2 weeks of 7 days) ♦ Solve number sentences within 99 involving addition or subtraction when one of the number quantities is unknown (e.g., 31 - ? = 18)	♦ Provide the range of solutions to inequalities in order to solve real-world problems when one of the number quantities is unknown ♦ Compare the magnitude of numbers on a number line using the symbols (<, =, >) ♦ Express an inequality to represent a real-world problem in one variable (e.g., I have 10 marbles, Tom has more marbles. Represent Tom's number of marbles; t > 10) ♦ Identify the missing number in equations involving multiplication ♦ Write an expression to represent a real-world situation using a variable for an unknown value ♦ Assess the reasonableness of answers to inequalities using mental computation

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Reason about and solve one-variable equations and inequalities. (continued)	♦ Respond to materials to add, subtract, and/or count known and unknown quantities and create graphs ♦ Orient materials used to add, subtract, and/or count known and unknown quantities (e.g., orient icon pictures used to label axis) ♦ Locate objects partially hidden or out of sight needed to add, subtract, and/or count known and unknown quantities (e.g., remove barrier to expose materials) ♦ Use one object to act on another to add, subtract, and/or count known and unknown quantities (e.g., use scissors to cut materials) ♦ Turn on technology used to add, subtract, and/or count known and unknown quantities (e.g., turn on technology tool to create graph program)	♦ Solve simple addition equations where the sum is represented by a variable (e.g., 5+7=r, what is r?) ♦ Simplify one-operation expressions involving addition and subtraction (e.g., 6 – 3; 12 + 9) ♦ Simplify one-operation expressions involving multiplication (e.g., 5 x 2; 6 x 4)	♦ Describe a context in which a sum is expressed (e.g., describe 6 + 5 as "I have six dollars and I found a five dollar bill") ♦ Solve simple multiplication equations where the product is represented by a variable (e.g., If 6 x 7=t, what is t?)	♦ Express number sentences to solve a real-world problem involving addition or subtraction when one of the number quantities is unknown *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ACCESS SKILLS (continued) for Expressions and Equations Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS **The student will:**	**ENTRY POINTS** **The student will:**
Reason about and solve one-variable equations and inequalities. (continued)	♦ Imitate action used to add, subtract, and/or count known and unknown quantities (e.g., imitate classmate attaching icon to graph) ♦ Initiate cause-and-effect response during adding, subtracting, and/or counting activity (e.g., turn on technology tool to activate counting computer program) ♦ Sustain adding, subtracting, and/or counting activity through response (e.g., use preprogrammed voice-generating device comment) ♦ Gain attention during adding, subtracting, and/or counting activity (e.g., raise hand, vocalize) ♦ Make a request during an activity to add, subtract, and/or count known and unknown quantities (e.g., request a turn) ♦ Choose from an array of two in an adding, subtracting, and/or counting activity (e.g., choose materials to be used in an adding activity)	

ENTRY POINTS and ACCESS SKILLS for Expressions and Equations Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Represent and analyze quantitative relationships between dependent and independent variables.	♦ Respond to materials to create graphs ♦ Attend visually, auditorially, or tactilely to materials to create graphs ♦ Track materials used to create graphs ♦ Shift focus from materials used to create graphs, to speaker ♦ Grasp materials used to create graphs ♦ Use two hands to hold materials used to create graphs ♦ Release materials used to create graphs ♦ Move materials used to create graphs ♦ Orient materials used to create graphs (e.g., orient icon pictures used to label axis) ♦ Manipulate objects used to create graphs ♦ Locate objects partially hidden or out of sight needed to create graphs (e.g., remove barrier to expose materials) ♦ Use one object to act on another to create graphs (e.g., use scissors to cut materials)	♦ Represent unknown number quantities in an input-output table, given an input number and a rule (e.g., input = 60, rule is subtract 15, then output = 45) ♦ Answer yes/no questions about input/output tables ♦ Generate a number pattern given an addition rule and an initial value (e.g., start with 6, rule is add 4, find the next 5 numbers in the pattern) ♦ Generate a number pattern given an subtraction rule and an initial value (e.g., start with 50, rule is subtract 7, find the next 5 numbers in the pattern)	♦ Determine the addition rule in an input-output table (e.g., if input is 20 and output is 25, what is the rule?) ♦ Determine the subtraction rule in an input-output table (e.g., if input is 60 and output is 45, what is the rule?) ♦ Generate a number pattern given an multiplication rule and an initial value (e.g., start with 2, rule is multiply by 3, find the next 4 numbers in the pattern)	♦ Express addition and subtraction equations to represent the relationship between two variables using graphs or tables (e.g., Input -15 = Output) ♦ Determine the multiplication rule in an input-output table (e.g., if input is 3 and output is 12, what is the rule?) ♦ Determine, given two variables, which is dependent and which is independent in a given situation (e.g., height of a student, and age of a student, which is the dependent variable?) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ACCESS SKILLS (continued) for Expressions and Equations Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	**ENTRY POINTS The student will:**
Represent and analyze quantitative relationships between dependent and independent variables. (continued)	♦ Turn on technology used to create graphs (e.g., turn on technology tool to create graph program) ♦ Imitate action used to create graphs (e.g., imitate classmate attaching icon to graph) ♦ Initiate cause-and-effect response during graphing activity (e.g., turn on technology tool to activate graphing computer program) ♦ Sustain graphing activity through response (e.g., use preprogrammed voice-generating device comment) ♦ Gain attention during graphing activity (e.g., raise hand, vocalize) ♦ Make a request during an activity to create graphs (e.g., request a turn) ♦ Choose from an array of two during graphing activity (e.g., choose materials to be used in graphing activity)	

CONTENT AREA Mathematics
DOMAIN Expressions and Equations

Grade 7

Cluster	Standards as written	
Use properties of operations to generate equivalent expressions.	**7.EE.A.1**	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
	7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	**7.EE.B.3**	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional **1/10** of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9¾ inches long in the center of a door that is 27½ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
	7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
	7.EE.B.4a	Solve word problems leading to equations of the form $px + q = r$ and $p(x \div q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
	7.EE.B.4b	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
	MA.7.EE.B.4c	Extend analysis of patterns to include analyzing, extending, and determining an expression for simple arithmetic and geometric sequences (e.g., compounding, increasing area), using tables, graphs, words, and expressions.

ENTRY POINTS for
Expressions and Equations Standards in Grade 7

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Use properties of operations to generate equivalent expressions.	♦ Match the associative, distributive and commutative properties of addition with the appropriate numerical expressions (e.g., match the term associative property to the expression (96 + 56) + 44 = 96 + (56 + 44)) ♦ Demonstrate the commutative property of addition using manipulatives *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Apply distributive properties to evaluate expressions involving one or more operations with positive and/or negative whole numbers (e.g., 6 x 8 + 6 x 2 = 6 (8 + 2)) ♦ Apply commutative property to expressions involving addition with positive and negative whole numbers (e.g., 21 + 9 = ? 21 + 9 = 9 + 21) ♦ Apply associative property to expressions involving addition with positive and negative whole numbers (e.g., 3 + (2 + 4) = ? 3 + (2 + 4) = (3 + 2) + 4 ♦ Match the associative property of multiplication with numerical expressions that exemplify the property (e.g., (6 x 4) x 10 = 6 x (4 x 10))	♦ Apply multiple properties to simplify expressions involving one or more operations with numbers and with letters representing unknown numbers given the value of the unknown number (e.g., 4(w - 3) + 2w = 4w – 12 + 2w = 4w + 2w – 12 = 6w - 12) ♦ Produce equivalent expressions by applying the distributive property to problems involving addition, subtraction, and multiplication (e.g., (10 x a) + (10 x b) = 10 (a + b)) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Expressions and Equations Standards in Grade 7

Less Complex → **More Complex**

	The student will:	The student will:	The student will:
Solve real-life and mathematical problems using numerical and algebraic expressions and equations	♦ Add and subtract simple fractions, using manipulatives or technology in the context of solving a real-world problem ♦ Multiply and divide simple fractions, using manipulatives or technology in the context of solving a real-world problem ♦ Multiply two-digit numbers by one-digit numbers using manipulatives and drawings in the context of solving a real-world problem ♦ Divide two-digit numbers by one-digit numbers using manipulatives and drawings in the context of solving a real-world problem ♦ Choose the number of objects within 20 that make a number sentence true using manipulatives or drawings in the context of solving a real-world problem ♦ Extend simple repeating arithmetic sequence (e.g., create a table to show the following situation: John makes $20 a week for mowing the lawn) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Add and subtract fractions with the same denominator using mixed numbers in the context of solving a real-world problem ♦ Multiply and divide fractions with the same denominator using mixed numbers in the context of solving a real-world problem ♦ Multiply three-digit numbers by one-digit numbers in the context of solving a real-world problem ♦ Divide two-digit numbers by one-digit numbers (no remainders) in the context of solving a real-world problem ♦ Add and subtract positive and/or negative numbers including decimals to tenths in the context of solving a real-world problem ♦ Multiply and divide positive and/or negative numbers including decimals to tenths in the context of solving a real-world problem ♦ Find the unknown number quantity within 10 that makes an equation true (e.g., x + 5 = 10)	♦ Solve real-world word problems involving adding and subtracting positive and/or negative fractions and mixed numbers ♦ Solve real-world word problems involving multiplying and dividing positive and/or negative fractions and mixed numbers ♦ Multiply up to three-digit numbers by two-digit numbers in the context of solving a real-world word problem ♦ Divide three-digit numbers by one-digit numbers (no remainders) in the context of solving a real-world word problem ♦ Add and subtract positive and/or negative numbers including decimals to hundredths in the context of solving a real-world word problem ♦ Determine if a given value represents the unknown number quantity in an equation (e.g., x - 12 = 24 for x = 12)) in the context of solving a real-world problem ♦ Solve for unknown numbers in a number sentence using addition/subtraction and /or multiplication/division *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Expressions and Equations

Grade 8

Cluster	Standards as written	
Work with radicals and integer exponents.	**8.EE.A.1**	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
	8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*
	8.EE.A.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Understand the connections between proportional relationships, lines, and linear equations.	**8.EE.B.5**	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
	8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
Analyze and solve linear equations and pairs of simultaneous linear equations.	**8.EE.C.7**	Solve linear equations in one variable.
	8.EE.C.7a	Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
	8.EE.C.7b	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the Distributive property and collecting like terms.
	8.EE.C.8	Analyze and solve pairs of simultaneous linear equations.

	8.EE.C.8a	Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
	8.EE.C.8b	Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*
	8.EE.C.8c	Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

ENTRY POINTS for
Expressions and Equations Standards in Grade 8

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Work with radicals and integer exponents.	♦ Match the figure of a square with given dimensions to determine its perfect square (e.g., match a square with sides of 5 to its perfect square 25) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Express the perfect square given a model of a square ♦ Evaluate numbers written with the exponent of two (e.g., $2^2 = 4$) ♦ Identify a geometric formula that represents the area of a square given the side lengths (e.g., $A = 4^2$) ♦ Illustrate the cube of a number (numbers written with the exponent of 3) with manipulatives (e.g., 10^3) ♦ Represent numbers in scientific notation	♦ Use the notation of exponents and square roots (√) to express the square roots of small perfect squares (e.g., 5 = sq root of 25; 5 squared = 25) ♦ Use exponents to indicate the number of repeated factors (e.g., $2 \times 2 \times 2 = 2^3$) ♦ Identify perfect squares within 100 ♦ Write (express) the equation that represents the area of a square and evaluate to find the total area in square units ♦ Identify perfect cubes within 1000 *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Understand the connections between proportional relationships, lines, and linear equations.	♦ Solve problems involving real-life using proportional relationships, including unit pricing (e.g., four apples cost 80 cents, so one apple costs 20 cents) ♦ Identify a specific data point when given the coordinates ♦ Use proportions to solve real-life problems involving unit rates (e.g., in Jasmine's homeroom, the ratio of girls to boys is 3:2. If there are 12 girls in Jasmine's homeroom, how many boys are there?) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Calculate a unit rate from a description of a relationship (e.g., if 2 pounds of apples cost $4.00, how much does 1 pound cost?) ♦ Plot points from a proportional relationship in the first quadrant of a coordinate plane ♦ Graph a simple proportional relationship when given the unit rate (2:1 converting to 2/1) ♦ Given an equation of a proportional relationship, determine the unit rate (slope) ♦ Given a graph of a proportional relationship, determine the unit rate (slope)	♦ Create graphs on a coordinate plane to represent a proportional relationship ♦ Compare two or more lines on a coordinate plane to determine whether the unit rate (slope) is > or < ♦ Given an equation of a proportional relationship, create a graph of the relationship ♦ Given an equation of a proportional relationship create a graph of the relationship ♦ Given a graph of a proportional relationship, express the equation of the relationship *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Expressions and Equations Standards in Grade 8

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Analyze and solve linear equations and pairs of simultaneous linear equations.	♦ Demonstrate the relationship between addition and subtraction as inverse operations (one undoes the other) using manipulatives or drawings ♦ Demonstrate the relationship between multiplication and division as inverse operations (one undoes the other) using manipulatives or drawings *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Solve for the unknown number in a number sentence using addition and subtraction (e.g., 16 - y = 7; 16 = 7 + y; 9 = y) ♦ Solve for the unknown number in a number sentence using multiplication and division (e.g., 16y = 48, 48/16 = y) ♦ Identify the inverse operation that can be used to solve for the unknown quantity in an algebraic equation involving addition or subtraction (e.g., x + 15 = 37 and 37 - 15 = x)	♦ Identify the inverse operation to solve for the unknown quantity in an algebraic equation using addition, subtraction, multiplication, and/or division *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Functions

	Standards	Entry Points	Access Skills
8	Page 107	Pages 108 – 110	Pages 108 – 111

CONTENT AREA Mathematics
DOMAIN Functions

Grade 8

Cluster	Standards as written	
Define, evaluate, and compare functions.	**8.F.A.1**	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[3]
	8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
	8.F.A.3	Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.*
Use functions to model relationships between quantities.	**8.F.B.4**	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
	8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

[3] Function notation is not required in grade 8.

ENTRY POINTS and ACCESS SKILLS for Functions Standards in Grade 8

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Define, evaluate, and compare functions.	♦ Respond to materials being compared by quantity or size ♦ Attend visually, auditorially, or tactilely to materials being compared by quantity or size ♦ Track materials being compared by quantity or size ♦ Shift focus from materials being compared by quantity or size to speaker ♦ Grasp materials being compared by quantity or size ♦ Release materials being compared by quantity or size ♦ Give materials being compared by quantity or size ♦ Move materials being compared by quantity or size ♦ Orient materials being compared by quantity or size ♦ Construct two objects using materials from two sets of materials being compared by quantity or size (e.g., build two block towers, with one set of 3 blocks and one with 5 blocks)	♦ Answer yes/no questions about functions ♦ Identify the input (x) and the output (y) in an input-output table and interpret the relationship between one variable and another variable in a table (e.g., number of hours, number of miles traveled) ♦ Create an input-output table when given the input values and the function rule ♦ Graph a function on the coordinate plane using a completed table ♦ Represent unknown number quantities in an input-output table, given an input number and a rule (e.g., input = 60, rule is subtract 15, then output = 45) ♦ Answer yes/no questions about input/output tables ♦ Generate a number pattern given an addition rule and an initial value (e.g., start with 6, rule is add 4, find the next 5 numbers in the pattern)	♦ Identify linear and non-linear functions from given tables by graphing them ♦ Explain why the relationships are or are not functions in given tables (e.g., given a table with x-input: 9, 9, 16, 16 and corresponding y-output: 3, -3, 4, -4 determine the relationship is not a function because some inputs have more than one output) ♦ Explain why the relationships are or are not functions in given graphs (e.g., a graph of a circle is not a function because for some x-inputs there is more than one y-output; a horizontal line on the coordinate plane is a function because for every x input there is only one y output) ♦ Determine the addition rule in an input-output table (e.g., if input is 20 and output is 25, what is the rule?)	♦ Explain why the relationships are or are not functions in given algebraic equations (e.g., the equation y=x is a function because each input has only one output and $y^2 + x^2 = 25$ is not a function because one input has 2 outputs) ♦ Compare *slopes* of two functions presented in different ways (e.g., the algebraic representation of the function y = 3x + 2 has a positive slope and the table representation of the function X input 0, 1, 2 and corresponding y outputs 2, 0, -2 has a negative slope) ♦ Determine whether a function is linear or non-linear by graphing it ♦ Describe how varying the rate of change in a variable affects the outcome in a table (e.g., increasing the speed decreases the time needed to arrive at a destination)

ENTRY POINTS and ACCESS SKILLS for Functions Standards in Grade 8

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Define, evaluate, and compare functions. (continued)	♦ Turn on voice-generating device to participate in an activity to compare materials by quantity or size ♦ Imitate action with materials being used to compare quantity or size ♦ Sustain activity comparing objects by size or quantity through response ♦ Gain attention in activity comparing objects by size and quantity (e.g., raise hand during comparison lesson on white board) ♦ Imitate action using materials compared by quantity or size ♦ Initiate cause-and-effect using materials compared by quantity or size ♦ Make a request in an activity comparing materials by size or quantity ♦ Choose from an array of two in an activity to compare materials by quantity ♦ Choose beyond an array of two to compare materials by quantity	♦ Generate a number pattern given an subtraction rule and an initial value (e.g., start with 50, rule is subtract 7, find the next 5 numbers in the pattern) ♦ Compare *initial values* of two functions presented in different ways (e.g.: Compare the initial value in Sam and Tom's savings account in the description and equations below: <u>description for Sam</u>: Sam deposits $20 in the bank to open an account. He puts $10 into the account each month and withdraws nothing. <u>Equation for Tom</u>: s= 4m+ 200 Answer: Sam's initial value is $20 and Tom's is $200. Tom has a greater initial value) ♦ Classify graphs of functions as linear or nonlinear given two graphical representations	♦ Determine the subtraction rule in an input-output table (e.g., if input is 60 and output is 45, what is the rule?) ♦ Generate a number pattern given an multiplication rule and an initial value (e.g., start with 2, rule is multiply by 3, find the next 4 numbers in the pattern) ♦ Compare rates of change of two functions presented in different ways (e.g.: Compare the rate of change in Sam and Tom's savings in the description and equations below. Description for Sam: Sam deposits $20 in the bank to open an account. He puts $10 into the account each month and withdraws nothing Equation for Tom: s= 4m+ 200 Answer: Sam puts $10 in his account every month and Tom puts $4 in his account every month so Sam's saving has a greater rate of change)	♦ Express addition and subtraction equations to represent the relationship between two variables using graphs or tables (e.g., Input -15 = Output) ♦ Determine the multiplication rule in an input-output table (e.g., if input is 3 and output is 12, what is the rule?) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS and ACCESS SKILLS for Functions Standards in Grade 8

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Use functions to model relation-ships between quantities.	♦ Respond to materials to create graphs ♦ Attend visually, auditorially, or tactilely to materials to create graphs ♦ Track materials used to create graphs ♦ Shift focus from materials used to create graphs to speaker ♦ Grasp materials used to create graphs ♦ Use two hands to hold materials used to create graphs ♦ Release materials used to create graphs ♦ Move materials used to create graphs ♦ Orient materials used to create graphs (e.g., orient icon pictures used to label axis) ♦ Manipulate objects used to create graphs ♦ Locate objects partially hidden or out of sight needed to create a graph (e.g., remove barrier to expose materials) ♦ Use one object to act on another to create graphs (e.g., use scissors to cut materials)	♦ Determine the initial value (y-intercept) and the rate of change (slope) from graphs ♦ Match given descriptions to a given graph (e.g., match segments on a graph to segments of the following description: Jenny is walking to Samantha's house at a constant rate. Jenny gets to Samantha's house and is waiting. Jenny and Samantha ride the bus to school. The bus is moving at a constant rate but much faster than Jenny's walking rate.) ♦ Answer yes/no questions about graphs	♦ Determine the initial value (y-intercept) from graphs and linear equations ♦ Determine the rate of change (slope) from graphs and linear equations ♦ Describe a graph of a function that has labeled sections (e.g., Describe that Part A rises gently, Part B is short but flat, Part C rises like part A, but is steeper, etc.)	♦ Determine the initial value (y-intercept) from tables, graphs, linear equations and descriptions ♦ Determine the rate of change (slope) from tables, graphs, linear equations and descriptions ♦ Sketch a graph from given context (e.g., given the context "Jenny is walking to Samantha's house at a constant rate" the student draws a straight line on the graph with a slope. And labels time on the x-axis and distance on the y-axis. When the context changes, "Jenny gets to Samantha's house and is waiting" the student draws a horizontal line, etc." *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ACCESS SKILLS (continued) for Functions Standards in Grade 8

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	ENTRY POINTS The student will:
Use functions to model relationships between quantities. (continued)	♦ Turn on technology used to create graphs ♦ Imitate action used to create graphs (e.g., imitate classmate attaching icon to graph) ♦ Initiate cause-and-effect response to turn on technology tool to activate graphing computer program ♦ Sustain graphing activity through response (e.g., using preprogrammed voice-generating device comment) ♦ Gain attention in a graphing activity (e.g., raise hand, vocalize) ♦ Make a request during a graphing activity (e.g., request a turn) ♦ Choose from materials to be used in graphing activity	

Measurement and Data

	Standards	Entry Points	Access Skills
PK	Page 113	Pages 114, 116, 118	Pages 114 – 118
K	Page 119	Page 120	
1	Page 121	Pages 122 – 123	Pages 122 – 123
2	Page 124	Pages 125 – 126	
3	Pages 127 – 128	Pages 129 – 130	
4	Page 131	Page 132	
5	Page 133	Page 134	

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Pre-Kindergarten

Cluster	Standards as written	
Describe and compare measurable attributes.	**MA.PK.MD.A.1**	Recognize the attributes of length, area, weight, and capacity of everyday objects using appropriate vocabulary (e.g., *long, short, tall, heavy, light, big, small, wide, narrow*).
	MA.PK.MD.A.2	Compare the attributes of length and weight for two objects, including longer/shorter, same length; heavier/lighter, same weight; holds more/less, holds the same amount.
Classify objects and count the number of objects in each category.	**MA.PK.MD.B.3**	Sort, categorize, and classify objects by more than one attribute.
Work with money.	**MA.PK.MD.C.4**	Recognize that certain objects are coins and that dollars and coins represent money.

ENTRY POINTS and ACCESS SKILLS for Measurement and Data Standards in Pre-K

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Describe and compare measure-able attributes.	♦ Respond to sets of materials presented by measurable attributes (e.g., big/small, long/short, heavier/lighter, less/more) ♦ Attend (visually, auditorially, or tactilely) to sets of materials labeled "same" or "different" ♦ Track materials that are labeled by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) ♦ Shift gaze from speaker to materials presented by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) ♦ Grasp materials presented by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more)	♦ Answer yes/no questions about subject matter about comparing measurable attributes of objects	♦ Identify an object by one measurable attribute (e.g., show me the big ball)	♦ Compare two objects based on length or width

ACCESS SKILLS (continued) for Measurement and Data Standards in Pre-K

Less Complex ← → **More Complex**

	ACCESS SKILLS The student will:	**ENTRY POINTS** The student will:
Describe and compare measurable attributes. (continued)	♦ Release materials presented by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) ♦ Give materials described by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) ♦ Move materials presented by measurable attribute (e.g., big/small, heavier/lighter, long/short, less/more) ♦ Manipulate objects with measurable attributes with two hands ♦ Locate partially hidden objects remove barrier between objects to compare measurable attributes ♦ Use one object to act on another ♦ Adjust planes to move objects with measurable attributes ♦ Make a request with the materials to be compared by measurable attribute ♦ Choose from an array of two to compare by measurable attribute with the presented object	

ENTRY POINTS and ACCESS SKILLS for Measurement and Data Standards in Pre-K

Less Complex ← → **More Complex**

	ACCESS SKILLS	ENTRY POINTS		
	The student will:	**The student will:**	**The student will:**	**The student will:**
Classify objects and count the number of objects in each category.	♦ Respond to materials as they are counted ♦ Attend visually, auditorially, or tactilely to materials as they are counted ♦ Shift focus from materials to speaker counting materials ♦ Grasp materials as they are counted ♦ Release materials as they are counted ♦ Give materials as they are counted ♦ Move objects as they are counted ♦ Orient objects as they are counted (e.g., turn flowerpots upright) ♦ Manipulate objects with two hands as they are counted ♦ Locate objects partially hidden or out of sight, to add or subtract to a collection of objects to be counted ♦ Use one object to act on another as objects are counted (e.g., use a pointer to tap) ♦ Adjust plane to move objects in counting activities (e.g., tip plank so that materials can be named in counting sequence as they fall)	♦ Collect objects with similar characteristics (e.g., all round objects)	♦ Sort items by one given measurable attribute (e.g., sort the pencils by length)	♦ Classify sorted objects based on measurable attribute (e.g., long and short pencils)

ACCESS SKILLS (continued) for Measurement and Data Standards in Pre-K

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	**ENTRY POINTS** The student will:
Classify objects and count the number of objects in each category. (continue)	♦ Construct using materials that have been counted in sequence (e.g., tower of blocks) ♦ Turn device on/off to participate in counting sequence activity (e.g., activate preprogrammed voice-generating device to recite number names) ♦ Imitate action in counting sequence activity ♦ Initiate cause-and-effect response in counting sequence activity (e.g., use switch to activate a number-naming cause-and-effect computer program) ♦ Gain attention in counting sequence activity ♦ Make a request in counting sequence Sustain counting sequence activity through response activity (e.g., request a turn to move the marker on a board game) ♦ Choose from an array of two during a counting sequence activity (e.g., choose materials to be counted) ♦ Choose beyond an array of two during a counting sequence activity (e.g., choose materials to be counted) ♦ Follow directions in counting sequence activities (e.g., follow direction to "Put the pencils in the box" as the teacher counts) ♦ Respond to materials as they are counted in sequence	

ENTRY POINTS and ACCESS SKILLS for Measurement and Data Standards in Pre-K

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Work with money.	♦ In the context of an academic activity, work with money ♦ Respond to bills and coins ♦ Attend visually, auditorially, or tactilely to bills and coins ♦ Track money ♦ Shift focus from money to speaker ♦ Grasp bills or coins ♦ Release bills or coins ♦ Give bills or coins ♦ Locate money partially hidden or out of sight (e.g., remove barrier to expose coins or bills) ♦ Adjust planes to move money ♦ Turn on/off (e.g., turn on prerecorded message about money on a voice-generating device) ♦ Imitate action (e.g., imitate a cash exchange) ♦ Initiate cause-and-effect response (e.g., initiate a cause-and-effect money computer program) ♦ Sustain a money activity through response ♦ Gain attention in an academic activity related to money ♦ Make a request in a money lesson (e.g., request a turn)	♦ Answer yes/no questions related to money		♦ Select a coin from other objects as something of value

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Kindergarten

Cluster	Standards as written	
Describe and compare measurable attributes.	**K.MD.A.1**	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
	K.MD.A.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*
Classify objects and count the number of objects in each category.	**K.MD.B.3**	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[4]

[4] Limit category counts to less than or equal to 10

ENTRY POINTS for Measurement and Data Standards in Kindergarten

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Describe and compare measurable attributes.		♦ Use measurable attributes to describe objects (e.g., the book is heavy, that shoe is long)	♦ Given two objects identify the object that is bigger
Classify objects and count the number of objects in each category.	♦ Count the number of presorted objects	♦ Count the number of objects with similar characteristics (e.g., given a mixed group of shapes, count the number of squares)	♦ Group each pre-sorted set by the number of objects in each set (e.g., both blue and green sets of buttons have 4)

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Grade 1

Cluster	Standards as written	
Measure lengths indirectly and by iterating length units.	**1.MD.A.1**	Order three objects by length; compare the lengths of two objects indirectly by using a third object.
	1.MD.A.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*
Tell and write time.	**1.MD.B.3**	Tell and write time in hours and half-hours using analog and digital clocks.
Represent and interpret data.	**1.MD.C.4**	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Work with money.	**MA.1.MD.D.5**	Identify the values of all U.S. coins and know their comparative values (e.g., a dime is of greater value than a nickel). Find equivalent values (e.g., a nickel is equivalent to 5 pennies). Use appropriate notation (e.g., 69¢). Use the values of coins in the solutions of problems.

ENTRY POINTS and ACCESS SKILLS for Measurement and Data Standards in Grade 1

Less Complex ⟷ **More Complex**

	ACCESS SKILLS	ENTRY POINTS		
	The student will:	**The student will:**	**The student will:**	**The student will:**
Measure lengths indirectly and by iterating length units.		♦ Order objects by length ♦ Use measurable attributes to describe 3 objects different in length. (e.g., the marker is l**onger** than the crayon but **shorter** than the pencil) ♦ Answer questions related to length and height with multiple objects (e.g., which is longer?) ♦ Identify from a group of similar objects which one is longest, tallest, widest, or deepest	♦ Tell how many units long an object is that has been premeasured with multiple copies of the shorter object (the length unit) end to end (e.g., this pencil is 6 paper clips long)	♦ Express the length of an object as a whole number of length units by laying multiple copies of the shorter object (the length unit) end to end (e.g., lay rectangular blocks end to end *without* overlaps or spaces until they reach the same length (side of a bookshelf)students should be given objects that match up exactly)
Tell and write time.	♦ Respond to daily schedule ♦ Attend (visually, auditorially, or tactilely) to daily schedule ♦ Track daily schedule materials as they are presented ♦ Shift gaze from speaker to daily schedule ♦ Grasp daily schedule pictures or icons ♦ Hold daily schedule pictures or icons ♦ Release daily schedule pictures or icons ♦ Give daily schedule pictures or icons ♦ Orient daily schedule pictures or icons	♦ Using daily schedule or other sequence of events, determine "now, later" or "first, then" ♦ Identify intervals of time (e.g., morning, afternoon, evening) ♦ Demonstrate understanding of the concepts today/tomorrow/ yesterday	♦ Sequence events using representation of daily schedule ♦ Tell time to the nearest hour using digital clocks	♦ Tell time to the nearest hour using analog clocks

ENTRY POINTS and ACCESS SKILLS for Measurement and Data Standards in Grade 1

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS	ENTRY POINTS		
	The student will:	**The student will:**	**The student will:**	**The student will:**
Tell and write time. (continued)	♦ Locate daily schedule pictures or icons partially hidden or out of sight ♦ Turn on/off time-monitoring device ♦ Gain attention to request a turn in an activity related to clocks ♦ Make a request in an activity in telling time ♦ Choose from an array of two choose materials in a clock or time activity			
Represent and interpret data.		♦ Organize objects or symbols collected by two characteristics on a template (e.g., girls' and boys' favorite ice cream flavor)	♦ Represent data using an organizational tool (e.g., chart, list, tally, table)	♦ Answer questions to interpret data that has been organized into categories (e.g., how many liked chocolate ice cream?)
Work with money.		♦ Identify U.S. coins ♦ Use appropriate notation to express the value of a coin or a combination of coins ♦ Identify coins needed for common purchases	♦ Identify the value of U.S. coins ♦ Find the sum of the values of a mixed group of coins ♦ Identify pennies by value ♦ Identify two U.S. coins by name ♦ Group and compare familiar objects (e.g., coins, quantities)	♦ Determine equivalent values of U.S. coins (e.g., five pennies equals one nickel) ♦ Solve simple problems involving the values of coins up to $0.99 ♦ Identify all U.S. coins by name and/or value ♦ Identify nickels and dimes by value

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Grade 2

Cluster	Standards as written	
Measure and estimate lengths in standard units.	**2.MD.A.1**	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
	2.MD.A.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
	2.MD.A.3	Estimate lengths using units of inches, feet, centimeters, and meters.
	2.MD.A.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
Relate addition and subtraction to length.	**2.MD.B.5**	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
	2.MD.B.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
Work with time and money.	**2.MD.C.7**	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
	MA.2.MD.C.7a	Know the relationships of time, including seconds in a minute, minutes in an hour, hours in a day, days in a week, a month, and a year; and weeks in a month and a year.
	2.MD.C.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*
Represent and interpret data.	**2.MD.D.9**	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
	2.MD.D.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems, using information presented in a bar graph.

ENTRY POINTS for Measurement and Data Standards in Grade 2

Less Complex ⟵ ⟶ **More Complex**

	The student will:	**The student will:**	**The student will:**
Measure and estimate lengths in standard units.	♦ Identify the correct tool to use given a situation requiring measurement *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Measure and compare the length of objects using a non-standard measuring device	♦ Measure length of objects using pre-selected standard tools (e.g., rulers, yardsticks, and meter sticks) ♦ Record and compare the measured length of objects using 2 pre-selected standard tools (rulers, yardsticks, and meter sticks) by repeating the use of the measurement tool/unit without gaps or overlaps (e.g., it takes a smaller number of **feet** to measure the height of the door than it does **inches**) ♦ Identify which one is longest, tallest, widest, or deepest from a group of similar objects using standard units of measurement ♦ Use two *different* standard units of measurement to describe an object (e.g., inches, feet, or yards) ♦ Estimate how many units of measurement are needed for a certain object *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Relate addition and subtraction to length.	*See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Solve addition and subtraction word problems within 20 involving lengths that are given in the same units using a visual model ♦ Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers (0, 1, 2.....) ♦ Represent whole number sums and differences within 20 on a number line	♦ Solve addition and subtraction word problems within 50 involving lengths that are given in the same units using a visual model ♦ Represent sums and differences of whole number lengths within 50 on a number line *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Measurement and Data Standards in Grade 2

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Work with time and money.	♦ Match digital time to analog time ♦ Identify $1, $5, and $10 dollar bills *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Tell time on analog to the nearest half hour ♦ Tell time on analog clock to the nearest half hour, including a.m. and p.m. ♦ Identify the value of a mixed array of bills and coins ♦ Express the value of money as dollars and cents	♦ Tell time on analog clocks to the nearest fifteen minutes ♦ Tell time on analog clocks to the nearest fifteen minutes, including a.m. and p.m. ♦ Solve simple problems involving the values of coins up to 99 cents *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Represent and interpret data.	♦ Record measurement data for multiple objects using a single unit scale ♦ Create a bar or picture graph to represent a data set *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Answer questions using measurement data represented on a bar or picture graph	♦ Generate length measurement data to the nearest inch using a ruler ♦ Construct a line plot using measurement data with multiple data points using whole numbers ♦ Answer questions using measurement data represented on a line plot ♦ Solve word problems using information presented on a bar or picture graph *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Grade 3

Cluster	Standards as written	
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	**3.MD.A.1**	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
	3.MD.A.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
Represent and interpret data.	**3.MD.B.3**	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
	3.MD.B.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.
Geometric measure-ment: understand concepts of area and relate area to multiplication and to addition.	**3.MD.C.5**	Recognize area as an attribute of plane figures and understand concepts of area measurement.
	3.MD.C.5a	A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
	3.MD.C.5b	A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
	3.MD.C.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
	3.MD.C.7	Relate area to the operations of multiplication and addition.
	3.MD.C.7a	Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
	3.MD.C.7b	Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
	3.MD.C.7c	Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the Distributive property in mathematical reasoning.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (continued)	**3.MD.C.7d**	Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.
Geometric measure-ment: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	**3.MD.D.8**	Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

ENTRY POINTS for Measurement and Data Standards in Grade 3

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Solve problems involving measure-ment and estimation of intervals of time, liquid volumes, and masses of objects.	♦ Determine intervals of time before and after a given time (e.g., what time is 1 hour before 3:05?) ♦ Choose the most appropriate measurement unit for an associated object (e.g., use grams to describe the weight of an apple) ♦ Choose the most appropriate tool to measure an associated object (e.g., use a tape measure for a pumpkin) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Tell time to the nearest five minutes on a digital and/or analog clock ♦ Use liters to express liquid volume measurements ♦ Use scales to measure mass in grams and kilograms ♦ Solve simple word problems involving addition and subtraction of time intervals of one or more hours	♦ Express time to the nearest minute on both a digital and analog clock ♦ Make estimates of liquid volume without measuring (e.g., is there enough water in the bucket to fill four liters?) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Represent and interpret data.	♦ Draw a scaled picture graph (e.g., each smiley face represents 5 kids) ♦ Answer questions about "more" and "less" using line plot data *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Draw a scaled bar graph (e.g., each square represents 5 pets)	♦ Answer questions based on a scaled picture and/or bar graph ♦ Generate length measurement data to the nearest half-inch using a ruler (measure and record) ♦ Construct a line plot with multiple measurement data points using a half-inch scale ♦ Answer questions based on line plot data *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Measurement and Data Standards in Grade 3

Less Complex ← → **More Complex**

Geometric measure-ment: understand concepts of area and relate area to multiplica-tion and to addition.	♦ Find the area of a rectangle by tiling an area and counting unit squares using manipulatives, technology, or visual models *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Find the area of rectangular by repeated addition of either columns or rows of tiles that comprise a rectangle (e.g., add tiles in a 3 by 4 rectangle to show that 3 + 3 + 3 + 3 = 12 square units or 4 + 4 + 4 = 12 square units)	♦ Find the area of rectangle by multiplying side lengths Continue to address skills and concepts that approach grade-level expectations in this cluster
Geometric measure-ment: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	*See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Measure the sides of a polygon to the nearest whole unit	♦ Calculate the perimeter of polygons Continue to address skills and concepts that approach grade-level expectations in this cluster

Grade 4

Cluster	Standards as written	
Solve problems involving measurement and conversion of measure-ments from a larger unit to a smaller unit.	**4.MD.A.1**	Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs* (*1, 12*), (*2, 24*), (*3, 36*), *…*
	4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
	4.MD.A.3	Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
Represent and interpret data.	**4.MD.B.4**	Make a line plot to display a data set of measurements in fractions of a unit (**1**/**2**, **1**/**4**, **1**/**8**). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
Geometric measurement: Understand concepts of angle and measure angles.	**4.MD.C.5**	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: (5a, 5b)
	4.MD.C.5a	An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through **1**/**360** of a circle is called a "one-degree angle," and can be used to measure angles.
	4.MD.C.5b	An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
	4.MD.C.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
	4.MD.C.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

ENTRY POINTS for Measurement and Data Standards in Grade 4

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	♦ Compare relative sizes of standard units of measurement by using manipulatives and record findings (e.g., record how to determine that a yard is equivalent to 3 feet by laying 3 rulers end–to-end next to a yard stick) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Measure and record the linear measurements of an object using two standard units of measure (e.g., measure a board first in centimeters and then millimeters) ♦ Measure and record liquid measurements using two standard units of measure (e.g., quarts and gallons) ♦ Measure and record time measurements using two standard units of measurement (e.g., hours and minutes)	♦ Solve word problems involving addition or subtraction for distances using a number line ♦ Solve word problems involving addition or subtraction for intervals of time using a number line ♦ Solve word problems involving addition or subtraction for liquid volume using a number line ♦ Solve word problems involving addition or subtraction for masses of objects using a number line ♦ Solve word problems involving addition or subtraction for money using a number line ♦ Solve area problems using visual models ♦ Solve perimeter problems using visual models *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Represent and interpret data.	*See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Generate length measurement data to the nearest ¼-inch using a ruler ♦ Construct a line plot with multiple data points using fractions	*Continue to address skills and concepts that approach grade-level expectations in this cluster*
Geometric measurement: Understand concepts of angle and measure angles.	♦ Measure angles using labeled circle wedges (e.g., 90 degree, 45 degree or 120 degree wedges) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate*	♦ Measure in whole number degrees angles using a protractor	♦ Sketch angles of a specified measure ♦ Solve addition and subtraction problems to find unknown angles *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Measurement and Data

Grade 5

Cluster	Standards as written	
Convert like measurement units within a given measurement system.	**5.MD.A.1**	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.
Represent and interpret data.	**5.MD.B.2**	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.	**5.MD.C.3**	Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
	5.MD.C.3a	A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
	5.MD.C.3b	A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
	5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
	5.MD.C.5	Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.
	5.MD.C.5a	Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the Associative property of multiplication.
	5.MD.C.5b	Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.
	5.MD.C.5c	Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

ENTRY POINTS for Measurement and Data Standards in Grade 5

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Convert like measurement units within a given measurement system.	♦ Match measurements to corresponding terms and abbreviations for length, weight, and capacity in units *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Convert units of measurement within the same system to solve real-world problems (e.g., convert cm to m)	♦ Use simple measurement conversions to solve real-world problems ♦ Convert among different sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Represent and interpret data.	♦ Use line plot data involving fractions to answer questions about "more" and "less" *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*		♦ Solve problems involving addition and subtraction of fractions by using information presented in line plots *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.	♦ Identify parts of a container to determine volume (base, height, length, width) ♦ Solve repeated addition problems about volume using technology or manipulatives (e.g., arrange 2 by 3 rectangles of cubes to show 6 + 6 + 6 = 18 cube units) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Calculate the volume of a rectangular container using manipulatives (e.g., "cubic units") ♦ Solve multiplication problems about volume using manipulatives or technology (e.g., arrange 2 by 3 rectangles of cubes to show 6 x 3=18 cube units)	♦ Solve real-world problems involving volume by building with cubes and applying volume formula *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Geometry

	Standards	Entry Points	Access Skills
PK	Page 136	Pages 137 – 138	Pages 131– 133
K	Page 139	Page 140	
1	Page 141	Page 141	
2	Page 142	Page 142	
3	Page 143	Page 143	
4	Page 144	Page 144	
5	Page 145	Page 146	
6	Page 146	Page 148	
7	Page 149	Page 149 – 150	
8	Page 151	Pages 152 – 153	

Pre-Kindergarten

Cluster	Standards as written	
Identify and describe shapes (squares, circles, triangles, rectangles).	**MA.PK.G.A.1**	Identify relative positions of objects in space, and use appropriate language (e.g., *beside, inside, next to, close to, above, below, apart*).
	MA.PK.G.A.2	Identify various two-dimensional shapes using appropriate language.
Analyze, compare, create, and compose shapes.	**MA.PK.G.B.3**	Create and represent three-dimensional shapes (ball/sphere, square box/cube, tube/cylinder) using various manipulative materials (such as popsicle sticks, blocks, pipe cleaners, pattern blocks).

ENTRY POINTS and ACCESS SKILLS for Geometry Standards in Pre-K

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Identify and describe shapes (squares, circles, triangles, rectangles).	♦ Respond to geometric shapes ♦ Attend visually, auditorially, or tactilely to geometric shapes ♦ Track geometric shapes ♦ Shift focus from geometric shapes to speaker ♦ Shift gaze from speaker to geometric shapes ♦ Grasp, release, and/or give geometric shapes ♦ Manipulate objects (e.g., move a cube from one hand to the other) ♦ Locate geometric shapes partially hidden or out of sight (e.g., remove barrier to expose geometric shapes) ♦ Use one object to act on another geometric shape (e.g., use a stapler to attach geometric shapes) ♦ Adjust planes to move geometric shapes	♦ Match simple identical two-dimensional shapes (e.g., squares, circles) ♦ Match identical three-dimensional shapes (e.g., ball/sphere, box/cube) using various manipulative materials	♦ Match similar shapes of different sizes (e.g., squares, circles) ♦ Demonstrate the relative positions of objects in space (e.g., beside, inside, next to, close to, above, below, apart) ♦ Match similar three-dimensional shapes of different sizes (e.g., ball/sphere, square box/cube, tube/cylinder) using various manipulative materials	♦ Identify simple shapes by name (e.g., circle, square, triangle) ♦ Use appropriate language to describe the relative positions of objects in space (e.g., beside, inside, next to, close to, above, below, apart)

ENTRY POINTS and ACCESS SKILLS for Geometry Standards in Pre-K

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		**ENTRY POINTS**	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Identify and describe shapes (squares, circles, triangles, rectangles). (continued)	♦ Turn device on/off to label geometric shapes, (e.g., camera to photograph geometric shapes in the environment) ♦ Imitate action in an activity using geometric shapes ♦ Initiate cause-and-effect response using geometric shapes ♦ Sustain geometry activity through response (e.g., when reading of geometry text stops, vocalize to request more) ♦ Gain attention (e.g., use communication strategy to request a turn or ask for help in an activity utilizing geometric shapes) ♦ Choose from an array of two geometric shapes			
Analyze, compare, create, and compose shapes.			♦ Compose simple two-dimensional shapes from manipulatives using a template (e.g., a square from popsicle sticks)	♦ Draw or compose simple two-dimensional shapes from manipulatives (e.g., a square from popsicle sticks, a circle from pipe cleaners)

CONTENT AREA Mathematics
DOMAIN Geometry

Kindergarten

Cluster	Standards as written	
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	**K.G.A.1**	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
	K.G.A.2	Correctly name shapes regardless of their orientations or overall size.
	K.G.A.3	Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
Analyze, compare, create, and compose shapes.	**K.G.B.4**	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
	K.G.B.5	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
	K.G.B.6	Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

ENTRY POINTS for Geometry Standards in Kindergarten

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	♦ Sort two- and three-dimensional shapes by attribute ♦ Compose a square by matching manipulatives to a square drawn on paper *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Identify three-dimensional shapes (e.g., ball/sphere, square box/cube, tube/cylinder) using various manipulative materials	♦ Differentiate shapes as either two-dimensional (lying in a plane, "flat") or three-dimensional ("solid") *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Analyze, compare, create, and compose shapes.	♦ Indicate one feature of a two-dimensional shape (e.g., round, straight) ♦ Match two shapes made from different materials ♦ Distinguish squares from circles using sets that are identical ♦ Distinguish three-dimensional shapes from two-dimensional shapes using manipulatives *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Given an attribute, find the two-dimensional shapes with that feature (e.g., find the shapes that have straight sides) ♦ Compose simple three-dimensional shapes from manipulatives (e.g., a sphere from clay or using technology) ♦ Distinguish squares from circles using sets that are not identical in size	♦ Given an attribute, find the two- or three-dimensional shapes with that feature ♦ Compose larger shapes from visual models of smaller simple shapes ♦ Compose simple two-dimensional shapes from manipulatives (e.g., a square from popsicle sticks or using technology) ♦ Distinguish squares, circles, and triangles using sets that are identical *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 1

Cluster	Standards as written	
Reason with shapes and their attributes.	**1.G.A.1**	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes that possess defining attributes.
	1.G.A.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape
	1.G.A.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

ENTRY POINTS for Geometry Standards in Grade 1

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Reason with shapes and their attributes.	♦ Match shapes to their defining attributes (e.g., find the shape that is round) ♦ Identify attributes of different shapes (e.g., which shape has three angles?) ♦ Sort objects by attributes such as color, shape, size *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Use appropriate language to describe shapes (straight lines, round, closed, three-sided, curved) ♦ Group objects by one attribute	♦ Identify defining differences between shapes ♦ Manipulate equal parts of a shape to compose a whole shape ♦ Partition (take apart) circles and rectangles into equal shares ♦ Group objects by two attributes (e.g., size and color) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 2

Cluster	Standards as written	
Reason with shapes and their attributes.	**2.G.A.1**	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
	2.G.A.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
	2.G.A.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

ENTRY POINTS for Geometry Standards in Grade 2

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Reason with shapes and their attributes.	♦ Create different two-dimensional shapes with four sides using templates ♦ Use appropriate terms to identify shapes (e.g., triangles, quadrilaterals, pentagons, hexagons, and cubes) ♦ Partition a rectangle into equal shares (e.g., horizontally and vertically) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Create different shapes with three sides ♦ Match shapes with their defining attributes (e.g., faces, angles) ♦ Match same shapes with different orientation ♦ Partition a circle into equal shares	♦ Create multiple different shapes with four unequal sides ♦ Use appropriate terms to define differences between shapes ♦ Sort simple shapes by attribute ♦ Partition circles and rectangles into equal shares *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Grade 3

Cluster	Standards as written	
Reason with shapes and their attributes.	**3.G.A.1**	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
	3.G.A.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal areas and describe the area of each part as ¼ of the area of the shape.*

ENTRY POINTS for Geometry Standards in Grade 3

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Reason with shapes and their attributes.	♦ Create two-dimensional shapes using manipulatives ♦ Compose two-dimensional shapes (e.g., polygons, circles, squares, triangles, rectangles) using templates ♦ Sort two-dimensional shapes by attributes (e.g., length of sides, number of sides) ♦ Compose and decompose rectangles into "halves," "thirds," and "quarters" using manipulatives ♦ Compose and decompose triangles to create different angles *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Create two-dimensional shapes (e.g., polygons, circles, squares, triangles, rectangles) from models or examples ♦ Sort three-dimensional shapes by attributes (e.g., length of sides, number of faces) ♦ Compose and decompose circles into "halves," "thirds," and "quarters" using manipulatives	♦ Create two-dimensional shapes (e.g., polygons, circles, squares, triangles, rectangles) given the name of the shape ♦ Compare shapes by describing their attributes ♦ Compose and decompose triangles, squares, rectangles to form other figures using manipulatives *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 4

Cluster	Standards as written	
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	**4.G.A.1**	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
	4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
	4.G.A.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

ENTRY POINTS for Geometry Standards in Grade 4

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	♦ Use appropriate terms to describe lines and angles (e.g., perpendicular, parallel, obtuse, acute) ♦ Label lines and angles ♦ Decompose two-dimensional objects by dividing them into identical halves using a line of symmetry *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Match lines and angles to their defining attributes (e.g., perpendicular, parallel, obtuse, acute) ♦ Sort lines and angles by two or more attributes ♦ Draw/create different angles (intersecting lines) from a model	♦ Identify differences between lines and angles ♦ Compare lines and angles by their attributes ♦ Identify angles within a shape (e.g., acute, obtuse, right) ♦ Distinguish between parallel and intersecting lines ♦ Identify linear geometric figures on a plane (points, rays/lines, line segments) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 5

Cluster	Standards as written	
Graph points on the coordinate plane to solve real-world and mathematical problems.	**5.G.A.1**	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
	5.G.A.2	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
Classify two-dimensional figures into categories based on their properties.	**5.G.B.3**	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
	5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties.

ENTRY POINTS for Geometry Standards in Grade 5

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Graph points on the coordinate plane to solve real-world and mathematical problems.	♦ Label components of the coordinate system (e.g., origin, ordered pair, x-axis) ♦ Plot numbers on a horizontal number line ♦ Determine the distance from the origin on a number line given a point on a horizontal number line *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Plot numbers on a vertical number line ♦ Determine the distance from the origin on a number line given a point on a vertical number line ♦ Determine the distance traveled on the x-axis from the origin given a point in a labeled coordinate plane	♦ Graph ordered pairs in the first quadrant of the coordinate plane ♦ Determine the distance traveled on the y-axis from the origin, given a point in a labeled coordinate plane ♦ Interpret real-world mathematical problems by plotting points on a number line *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Classify two-dimensional figures into categories based on their properties.	♦ Identify given shapes presorted into subcategories determine what they have in common (e.g., rectangles, parallelograms and trapezoids are all four-sided shapes) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*		♦ Classify quadrilaterals in a hierarchy based on properties *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 6

Cluster	Standards as written	
Solve real-world and mathematical problems involving area, surface area, and volume.	**6.G.A.1**	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
	MA.6.G.A.1a	Use the relationships among radius, diameter, and center of a circle to find its circumference and area.
	MA.6.G.A.1B	Solve real-world and mathematical problems involving the measurements of circles.
	6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = lwh and V = bh to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
	6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
	6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface areas of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

ENTRY POINTS for Geometry Standards in Grade 6

Less Complex ← → **More Complex**

	The student will:	The student will:	The student will:
Solve real-world and mathematical problems involving area, surface area, and volume.	♦ Find the area of parallelograms by decomposing into rectangles and triangles ♦ Identify radius, diameter, and center of a circle ♦ Identify shapes that would measure volume (cone, sphere, cylinder) ♦ Trace all of the faces of a three-dimensional figure to show the surface area ♦ Construct a cube with whole side lengths using unit cubes of half side lengths (e.g., if one side of a unit cube has a length of one-half, it will take 8 unit cubes to make one whole cube) ♦ Calculate the area of a figure using manipulatives ♦ Calculate the area of a figure given formula *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Find the area of triangles by composing into rectangles ♦ Using a straightedge or technology draw the radius and the diameter through the center of a circle ♦ Match a three dimensional figure to the appropriate net made of rectangles and triangles ♦ Find the volume of a right rectangular prism with fractional side lengths of halves using unit cubes whose side lengths are designated as a half (e.g., a right rectangular prism of side lengths 4½ by 2 by 3 would be packed with 9 by 4 by 6 unit cubes of side lengths of a half) ♦ Calculate the surface area of a rectangular prism using manipulatives (e.g., by counting the number of unit squares contained in each face) ♦ Solve real-life problems for area using the formula	♦ Find the area of triangles and parallelograms by composing into rectangles ♦ Find the area of triangles and parallelograms by decomposing into rectangles ♦ Find the area of triangles or parallelograms in real-world situations by composing and/or decomposing into rectangles ♦ Calculate the circumference of circles, given the radius ♦ Calculate the area of circles given the radius ♦ Find the volume of right rectangular prisms with fractional side lengths in two ways, by building with cubes and using the volume formula ♦ Plot coordinates to find the length of a line joining points with the same first coordinate or the same second coordinate (e.g., plotting (2,3) and (2,7) shows a vertical line that is 4 units long) ♦ Match a net made up of rectangles and triangles to its three-dimensional shape and use it to find the surface area *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Geometry

Grade 7

Cluster	Standards as written	
Draw, construct, and describe geometrical figures and describe the relationships between them.	**7.G.A.1**	Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
	7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
	7.G.A.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	**7.G.B.4**	Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
	7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure.
	7.G.B.6	Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
	MA.7.G.B.7	Solve real-world and mathematical problems involving the surface area of spheres.

ENTRY POINTS for Geometry Standards in Grade 7

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Draw, construct, and describe geometrical figures and describe the relationships between them.	♦ Construct geometric shapes with given conditions (e.g., a shape with 4 right angles and two sides that are two inches, and two sides that are one inch long) ♦ Using premeasured line segment manipulatives (e.g., pipe cleaners) construct as many triangles as possible *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Construct triangles from three measures of sides ♦ Using premeasured line segment manipulatives construct as many triangles as possible and draw them (e.g., pipe cleaners) ♦ Reproduce a two-dimensional geometric figure at a different scale, using a straightedge, compass, computer software, and/or other tools	♦ Compute actual lengths and areas from a scale drawing ♦ Construct triangles from three measures of angles or sides ♦ Construct as many triangles as possible, using premeasured line segment manipulatives (e.g., pipe cleaners) ♦ Record which three side-length combinations do not form triangles *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	♦ Identify two pairs of vertical angles and four pairs of supplementary angles given vertical angles labeled a, b, c, d *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Solve an equation to find the missing angle measure when given one angle measure and told whether the adjacent angles are supplementary or complementary	♦ Solve real-world problems for area using the formula ♦ Solve real-world problems for circumference using the formula ♦ Write and solve an equation to find the missing angle measure when given one angle measure and told whether the non-adjacent angles are supplementary or complementary ♦ Given one measure of vertical angles, find the missing measures ♦ Calculate the surface area of a cube in real world situations ♦ Calculate the surface area of a rectangular prism in real world situations ♦ Calculate the volume of rectangular prisms given the measures of the length, width, and height in real world situations ♦ Calculate the surface area of a sphere in real world situations *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Grade 8

Cluster	Standards as written	
Understand congruence and similarity using physical models, transparencies, or geometry software.	**8.G.A.1**	Verify experimentally the properties of rotations, reflections, and translations: (1a, 1b, 1c)
	8.G.A.1a	Lines are taken to lines, and line segments to line segments of the same length.
	8.G.A.1b	Angles are taken to angles of the same measure.
	8.G.A.1c	Parallel lines are taken to parallel lines.
	8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
	8.G.A.3	Describe the effects of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
	8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
	8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
Understand and apply the Pythagorean Theorem.	**8.G.B.6**	Explain a proof of the Pythagorean Theorem and its converse.
	8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
	8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	**8.G.C.9**	Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems.

ENTRY POINTS for Geometry Standards in Grade 8

Less Complex ← → **More Complex**

	The student will:	The student will:	The student will:
Understand congruence and similarity using physical models, trans-parencies, or geometry software.	♦ Demonstrate a reflection, translation, and/or rotation of a two-dimensional figure using manipulatives ♦ Demonstrate a reflection, translation, and/or rotation of a two-dimensional figure using manipulatives and/or technology on a coordinate plane ♦ In a coordinate plane reflect a figure across the y- or x-axis and show congruence by tracing or placing a transparency over both figures to show that they are the same ♦ Record movement resulting from reflections, rotations or translations by counting to measure how many spaces each of the vertices of a figure moved left, right, up or down on a coordinate plane ♦ Demonstrate which angles are congruent by placing angle wedges in a diagram showing two parallel lines cut by a traversal and comparing those wedges *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ In a coordinate plane translate a figure a given number of space up, down, left or right and show congruence by tracing or placing a transparency over both figures to show that they are the same ♦ Determine whether triangles are similar given their side measures, angles, and shape (Note: similar means same size angles, but different side lengths) ♦ Use manipulatives or technology to find the sum of the interior angles of triangles of various shapes ♦ Using knowledge of supplementary angles to find the missing angle measure when one is given in a diagram showing two parallel lines cut by a traversal	♦ In a coordinate plane rotate a rectangle 90 degrees about the origin and show congruence by tracing or placing a transparency over both figures to show that they are the same ♦ Demonstrate a sequence that exhibits the similarity between two given rectangles on a coordinate plane ♦ Use manipulatives or technology to find the measures of the exterior angles of various shapes ♦ Using knowledge of vertical angles to find missing angle measure when one is given in a diagram showing two parallel lines cut by a traversal *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Geometry Standards in Grade 8

Less Complex ← → **More Complex**

Understand and apply the Pythagorean Theorem.	♦ Label the components of a right triangle (e.g., hypotenuse, legs, right angle) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Apply the Pythagorean Theorem to find the length of the hypotenuse on a right triangle on a coordinate plane	♦ Use models to show a relationship between the sum of the areas of the squares formed by the legs of a right triangle and the area of the square formed by its hypotenuse ♦ Apply the Pythagorean Theorem in real world situations to find a missing length in a naturally occurring right triangle (e.g., length of a ladder) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	*See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Use formulas to calculate the volume of a cylinder in real-life and/or mathematical problems	♦ Use formulas to calculate the volume of a sphere in real-life and/or mathematical problems ♦ Use formulas to calculate the volume of a cone in real-life and/or mathematical problems *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Statistics and Probability

	Standards	Entry Points	Access Skills
6	Page 155	Page 156–157	Pages 156 – 158
7	Pages 159 – 160	Pages 161– 162	
8	Page 163	Page 164	

Grade 6

Cluster	Standards as written	
Develop understanding of statistical variability.	**6.SP.A.1**	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
	6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
	6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
Summarize and describe distributions.	**6.SP.B.4**	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
	MA.6.SP.B.4a	Read and interpret circle graphs.
	6.SP.B.5	Summarize numerical data sets in relation to their context, such as by: (5a, 5b, 5c, 5d)
	6.SP.B.5a	Reporting the number of observations.
	6.SP.B.5b	Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
	6.SP.B.5c	Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
	6.SP.B.5d	Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

ENTRY POINTS and ACCESS SKILLS for Statistics and Probability Standards in Grade 6

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Develop understanding of statistical variability.	♦ Turn on technology used to collect data (e.g., turn on computer to gather information) ♦ Imitate action used to conduct survey (e.g., imitate classmate to distribute survey) ♦ Sustain a survey activity through response (e.g., reach out to shake hands to approach a survey respondent) ♦ Gain attention to conduct a survey (e.g., raise hand, vocalize)	♦ Identify the "center" and the "spread" of data represented graphically	♦ Find the median in a data set that has been displayed	♦ Sort questions that are statistical from those that are not (e.g., "How old are the students in my school?" is statistical, "How old am I?" is not) ♦ Identify the mean of a given set of data
Summarize and describe distributions.	♦ Respond to materials to create tables, charts, or graphs ♦ Attend visually, auditorially, or tactilely to materials to create tables, charts, or graphs ♦ Track materials used to create tables, charts, or graphs ♦ Shift focus from materials used to create tables, charts, or graphs to speaker ♦ Grasp materials used to create tables, charts, or graphs ♦ Use two hands to hold materials used to create tables, charts, or graphs	♦ Collect the responses to a survey ♦ Describe the spread (range) of a given set of data ♦ Match the given data displays to their names (e.g., match a data display histogram to the word histogram)	♦ Represent the data from a survey graphically (e.g., table, chart, manipulatives, tally, dot plot, histogram, box plot) ♦ Interpret data in a table, chart, or circle graph by answering questions related to the data ♦ Identify the mean of a set of data ♦ Match given data displays to given numerical data sets ♦ Match given data displays to given observations	♦ Select the most appropriate graphical representations for displaying a set of data (e.g., choose between a dot plot, a histogram, or a box plot and explain why that representation is most appropriate for a given set of data) ♦ Identify the spread and measures of center (mean and median) of a data set

ENTRY POINTS and ACCESS SKILLS for Statistics and Probability Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	The student will:	ENTRY POINTS The student will:	The student will:
Summarize and describe distributions. (continued)	♦ Release materials used to create tables, charts, or graphs ♦ Move materials used to create tables, charts, or graphs ♦ Orient materials used to create tables, charts, or graphs ♦ Manipulate objects used to create tables, charts, or graphs ♦ Locate objects partially hidden or out of sight needed to create tables, charts, or graphs (e.g., remove barrier to expose materials) ♦ Use one object to act on another to create tables, charts, or graphs (e.g., use scissors to cut materials) ♦ Turn on/off technology used to create tables, charts, or graphs (e.g., turn on voice-generating device to turn on technology tool to create graph program) ♦ Imitate action used to create tables, charts, or graphs (e.g., imitate classmate attaching icon to graph)		♦ Label parts of a given data display (e.g., for a given box plot, locate the minimum, quartile 1, median, quartile 3, and maximum)	♦ State observations from data displays (e.g., from histogram graph of minutes spent doing homework and number of students: a student might state "from the peak of the graph I can tell that most people do homework for 30-40 minutes" or "hardly anyone does homework for more than 160 minutes") ♦ Describe the measures of center for data sets from graphic representations ♦ Describe the measures of variability for data sets represented by graphical displays

ACCESS SKILLS (continued) for Statistics and Probability Standards in Grade 6

Less Complex ⟷ **More Complex**

	ACCESS SKILLS **The student will:**	**ENTRY POINTS** **The student will:**
Summarize and describe distributions. (continued)	♦ Initiate cause-and-effect response by turning on technology tool to activate graphing computer program ♦ Sustain graphing activity through response ♦ Gain attention in a graphing activity (e.g., raise hand, vocalize) ♦ Make a request during graphing relationship activity (e.g., request a turn) ♦ Choose materials to be used in graphing activity	

CONTENT AREA Mathematics
DOMAIN Statistics and Probability

Grade 7

Cluster	Standards as written	
Use random sampling to draw inferences about a population.	**7.SP.A.1**	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
	7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
Draw informal comparative inferences about two populations.	**7.SP.B.3**	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
	7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
Investigate chance processes and develop, use, and evaluate probability models.	**7.SP.C.5**	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
	7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
	7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
	7.SP.C.7a	Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

Investigate chance processes and develop, use, and evaluate probability models. (continued)	**7.SP.C.7b**	Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
	7.SP.C.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
	7.SP.C.8a	Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
	7.SP.C.8b	Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
	7.SP.C.8c	Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

ENTRY POINTS for Statistics and Probability Standards in Grade 7

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Use random sampling to draw inferences about a population.	♦ Interpret graphical representations of data (e.g., number of people in sample, information collected) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Draw conclusions from a data sample represented graphically (e.g., least/ greatest)	♦ Explain how a random survey sample could be obtained from a larger population set (e.g., every other person vs. every fifth person in a class list) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Draw informal comparative inferences about two populations.	♦ Interpret two sets of numerical data (e.g., there are more players on the football team (set 1) than there are on the basketball team (set 2)) *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Draw conclusions from the comparison of two sets of numerical data (e.g., more students have brown eyes than blue eyes) ♦ Compare data from a local survey to a published norm (e.g., class distribution of men vs. women to national statistics)	♦ Draw conclusions from the comparison of two sets of numerical data using measures of center (e.g., mean height of members of the football team and members of the basketball team) ♦ Compare data from a random sample vs. a non-random sample (e.g., compare the selection of favorite songs of a random sample of the school vs. your best friends) ♦ Determine the most accurate measure of center from examining data sets *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Investigate chance processes and develop, use, and evaluate probability models.	♦ Determine that the probability of an event occurring is likely or unlikely ♦ Record the outcomes in a simple probability experiment (e.g., choosing a red marble from a bag of assorted marbles) ♦ Use models to calculate the probability of an event (e.g., probability of landing on the green section of a spinner)	♦ Determine that the probability of an event occurring is likely, unlikely, certain, or impossible ♦ Interpret the outcomes of a probability experiment with different number of trials ♦ Explain predictions of the outcomes of a probability experiment (e.g., getting an even number on a number cube)	♦ Describe the probability of an event occurring as a number between 0 (impossible) and 1 (certain) ♦ Compare actual results of a simple experiment with theoretical probabilities using models and tables

ENTRY POINTS for
Statistics and Probability Standards in Grade 7

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Investigate chance processes and develop, use, and evaluate probability models. (continued)	*See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Select possible outcomes of a compound event from cards (e.g., if footwear (sneakers, flats, heels) is available in red and blue, select from cards that give pictures of all possible outcomes; i.e., heels in red, heels in blue, sneakers in red, sneakers in blue, flats in red, flats in blue)	♦ Organize (list/show/draw) all possible outcomes for compound events (e.g., if footwear: sneakers, flats and heels is available in colors: red and blue, list all possible selections – sneakers in red, flats in red, heels in red, sneakers in blue, heals in blue, flats in blue) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
DOMAIN Statistics and Probability

Grade 8

Cluster	Standards as written	
Investigate patterns of association in bivariate data.	**8.SP.A.1**	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
	8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
	8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
	8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

ENTRY POINTS for
Statistics and Probability Standards in Grade 8

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Investigate patterns of association in bivariate data.	♦ Represent numerical data on a scatter plot ♦ Identify the bivariate data represented by points on a scatter plot (e.g., age, height) ♦ Describe the presence or absence of a trend on a scatter plot *See entry points for earlier grades in this or a related cluster that are challenging and use age-appropriate materials*	♦ Describe a pattern of points on a scatter plot using terminology (e.g., "increasing to the right," "decreasing to the right," "neither," "positive correlation," "negative correlation," "no correlation," or "no pattern") ♦ Analyze frequencies in a two-way table (e.g., do students who have pets also check out books about animals from the library?) ♦ Find line of best fit for a scatter plot ♦ Complete a partially filled two way table summarizing data on two categorical variables from the same subject (e.g., fill in missing data in a table presenting survey data from 20 people asking if they had dogs <u>and</u> if they had allergies)	♦ Determine whether points on a scatter plot have a linear association (i.e., can a line be drawn that comes closest to most data points) ♦ Interpret the relationship between two variables in a scatter plot when there is a linear relationship (e.g., math scores vs. number of days students were absent) ♦ Use the line of best fit on a scatter plot of real-life data to predict likely outcomes (e.g., plant's growth, inches of rainfall) ♦ Interpret slope from linear models of bivariate data (e.g., from the temperature and ice cream sales graph, tell that ice cream sales go up as temperatures go up) ♦ Interpret y-intercept from linear models of bivariate data (e.g., for a gym that charges a yearly membership of $100, tell from the given negatively correlated linear model: gym visit (#) and cost ($) of visit, that the y-intercept is membership fee.) ♦ Interpret a given two-way table summarizing data on two categorical variables from the same subject (e.g., from data surveying 20 people asking if they had dogs <u>and</u> if they had allergies) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

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High School Mathematics *Number and Quantity – The Real Number System*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Number and Quantity
DOMAIN The Real Number System

High School

Cluster	Standards as written	
Extend the properties of exponents to rational exponents.	**H.N-RN.1**	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
	H.N-RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Use properties of rational and irrational numbers.	**H.N-RN.3**	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

ENTRY POINTS and ACCESS SKILLS for Number and Quantity
The Real Number System Standards in High School

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Extend the properties of exponents to rational exponents.	♦ Respond to materials as they are counted ♦ Attend visually, auditorially, or tactilely to materials as they are counted ♦ Shift focus from materials to speaker counting materials ♦ Grasp materials as they are counted ♦ Release materials as they are counted ♦ Give materials as they are counted ♦ Move objects as they are counted ♦ Orient objects as they are counted (e.g., turn flowerpots upright) ♦ Manipulate objects with two hands as they are counted ♦ Locate objects partially hidden or out of sight to add to a collection of objects to be counted ♦ Use one object to act on another as objects are counted (e.g., use a pointer to tap)	♦ Create a model of a perfect square	♦ Rewrite expressions with whole numbers using whole number exponents (e.g., $2 \times 2 \times 2 = 2^3$) ♦ Match the figure of a square with given dimensions to determine its perfect square (e.g., match a square with sides of 5 units to its area of 25 square units) ♦ Evaluate numbers written with the exponent of two (e.g., $2^2 = 4$) ♦ Identify a geometric formula that represents the area of a square given the side lengths (e.g., $A = 4^2$) ♦ Illustrate the cube of a number (numbers written with the exponent of 3) using manipulatives (e.g., 10^3) ♦ Express the square roots of perfect squares, using the notation of exponents and square roots ($\sqrt{}$) to (e.g., $5 = \sqrt{25}$; $5^2 = 25$) ♦ Identify perfect squares within 100	♦ Express the square roots of imperfect squares, using radical notation ($\sqrt{}$) ♦ Express the equation that represents the area of a square using exponents (e.g., $6^2 = 36$) ♦ Identify perfect cubes within 1000 ♦ Represent numbers in scientific notation (e.g., $432 = 4.32 \times 10^2$) ♦ Solve problems involving square roots (e.g., the area of a square is 20; estimate the length of a side) ♦ Estimate square roots of rational numbers *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ACCESS SKILLS (continued) for Number and Quantity
The Real Number System Standards in High School

Less Complex ⟷ **More Complex**

	ACCESS SKILLS **The student will:**	**ENTRY POINTS** **The student will:**
Extend the properties of exponents to rational exponents. (continued)	♦ Functionally use materials in a counting activity ♦ Locate partially hidden or out of sight objects or materials in a counting activity ♦ Construct or assemble materials in a counting activity ♦ Activate device in a counting activity ♦ Imitate action in a counting activity ♦ Initiate cause and effect response in a counting activity ♦ Sustain activity through response in a counting activity ♦ Gain attention in a counting activity ♦ Make a request in a counting activity ♦ Choose from an array of errorless choices in a counting activity ♦ Use one object to act on another in a counting activity (e.g., use a pointer to tap)	

ENTRY POINTS and ACCESS SKILLS for Number and Quantity
The Real Number System Standards in High School

Less Complex ⟷ **More Complex**

	ACCESS SKILLS	**ENTRY POINTS**	
	The student will:	**The student will:**	**The student will:**
Use properties of rational and irrational numbers.	(Refer to Access Skills above)	♦ Identify parts of a whole using visual fraction models and/or objects ♦ Record results of the comparison of two fractions using symbols (<, =, >) ♦ Create visual representation of fractions ♦ Label fractions on a number line ♦ Add and/or subtract fractions using manipulatives, visual models and/or technology (e.g., showing that two halves equal a whole or two fourths equal a half) ♦ Identify numbers that can be divided by 2	♦ Order fractions on a number line ♦ Add and/or subtract fractions with unlike denominators	♦ Multiply and/or divide fractions ♦ Identify irrational numbers ♦ Categorize rational and irrational numbers ♦ Estimate the placement of irrational numbers on a number line ♦ Determine if a number is rational or irrational by determining if it is (or can be written as) a fraction or a repeating or terminating decimal (e.g., 2.33333…, 3.567, $\sqrt{25}$, but not $\sqrt{10}$) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Number and Quantity
DOMAIN Quantities

High School Mathematics *Number and Quantity – Quantities*

High School

Cluster	Standards as written	
Reason quantitatively and use units to solve problems.	**H.N-Q.1**	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ⋆
	H.N-Q.2	Define appropriate quantities for the purpose of descriptive modeling. ⋆
	H.N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ⋆
	H.N-Q.MA3a	Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure. ⋆

⋆ indicates Modeling standard

ENTRY POINTS and ACCESS SKILLS for Number and Quantity Quantities Standards in High School

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		**ENTRY POINTS**	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Reason quantitatively and use units to solve problems.	♦ Attend visually, auditorially, or tactilely to materials as they are counted or measured ♦ Shift focus from materials to speaker counting materials or measured ♦ Grasp materials as they are counted or measured ♦ Release materials as they are counted or measured ♦ Give materials as they are counted or measured ♦ Move objects as they are counted or measured ♦ Orient objects as they are counted (e.g., turn flowerpots upright) or measured ♦ Manipulate objects with two hands as they are counted or measured ♦ Locate objects partially hidden or out of sight to add or subtract to a collection of objects to be counted or measured	♦ Describe situations in which two quantities have a relationship (e.g., rate = distance / time) ♦ Compare part-to-whole relationships (e.g., how many goldfish, how many total fish?) ♦ Compare part-to-part relationships (e.g., how many guppies, how many goldfish?) ♦ Create ratios among objects sorted by attribute ♦ Estimate how many units of measurement are needed for a certain object ♦ Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	♦ Determine a ratio, given real-world problems. ♦ Create a model that represents the relationship of two given quantities, using drawings, manipulatives, or technology ♦ Determine a unit rate, given a real-world problem ♦ Use rounding strategies to make estimates ♦ Choose the appropriate unit of measurement for a real-life problem (e.g., do you use grams or kilograms to describe the weight of a pig?) ♦ Choose an appropriate scale for a data display (e.g., scatterplot where each square represents 100 units)	♦ Perform conversions of units between systems or scales of measurement ♦ Solve proportions in real-life situations where one quantity is unknown (e.g., if you can buy 3 avocados for $6, how many can you buy for $18?) ♦ Express the solution to a real-life measurement problem using the appropriate unit (e.g., inches, feet, yard, or miles) ♦ Draw conclusions from comparative unit rates *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ACCESS SKILLS (continued) for Number and Quantity Quantities Standards in High School

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	**ENTRY POINTS The student will:**
Reason quantitatively and use units to solve problems. (continued)	♦ Use one object to act on another as objects are counted or measured (e.g., use a pointer to tap, hold measurement tool against object to be measured) ♦ Adjust plane to move objects in counting activities or measurement (e.g., tip plank so that materials can be named in counting sequence as they fall) ♦ Construct using materials that have been counted in sequence or measured (e.g., tower of blocks) ♦ Turn device on/off to participate in counting sequence or measurement activity (e.g., activate preprogrammed voice-generating device to recite number names) ♦ Imitate action in counting sequence or measurement activity ♦ Initiate cause-and-effect response in counting sequence or measurement activity (e.g., use switch to activate a number-naming cause-and-effect computer program) ♦ Sustain counting sequence activity through response ♦ Gain attention in counting sequence or measurement activity ♦ Make a request in counting sequence or measurement activity (e.g., request a turn to move the marker on a board game) ♦ Choose from an array of two during a counting sequence or measurement activity (e.g., choose materials to be counted or measured) ♦ Choose beyond an array of two during a counting sequence or measurement activity (e.g., choose materials to be counted or measured) ♦ Follow directions in counting sequence or measurement activities (e.g., follow direction to "Put the pencils in the box" as the teacher counts) ♦ Track object as it is added or subtracted from a set of objects	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Number and Quantity
DOMAIN The Complex Number System

High School Mathematics *Number and Quantity – The Complex Number System*

High School

Cluster	Standards as written	
Perform arithmetic operations with complex numbers.	**H.N-CN.1**	Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real.
	H.N-CN.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
	H.N-CN.3	(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Represent complex numbers and their operations on the complex plane.	**H.N-CN.4**	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
	H.N-CN.5	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}i)^3 = 8$ *because* $(-1 + \sqrt{3}i)$ *has modulus 2 and argument 120°.*
	H.N-CN.6	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
Use complex numbers in polynomial identities and equations.	**H.N-CN.7**	Solve quadratic equations with real coefficients that have complex solutions.
	H.N-CN.8	(+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$.
	H.N-CN.9	(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

[+] indicates standard is beyond College and Career Ready

Less Complex **More Complex**

Standards labeled with a (+) symbol appear in courses intended for all students, but are considered "beyond College and Career Ready" and will not be assessed.

High School Mathematics *Number and Quantity – Vector and Matrix Quantities*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Number and Quantity
DOMAIN Vector and Matrix Quantities

High School

Cluster	Standards as written	
Represent and model with vector quantities.	**H.N-VM.1**	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lVert\boldsymbol{v}\rVert$, v).
	H.N-VM.2	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
	H.N-VM.3	(+) Solve problems involving velocity and other quantities that can be represented by vectors.
Perform operations on vectors.	**H.N-VM.4**	(+) Add and subtract vectors.
	H.N-VM.4a	(+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
	H.N-VM.4b	(+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
	H.N-VM.4c	(+) Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
	H.N-VM.5	(+) Multiply a vector by a scalar.
	H.N-VM.5a	(+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
	H.N-VM.5b	(+) Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lVert c\boldsymbol{v}\rVert = \lvert c\rvert\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\lvert c\rvert\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

[+] indicates standard is beyond College and Career Ready

Perform operations on matrices and use matrices in applications.	**H.N-VM.6**	(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
	H.N-VM.7	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
	H.N-VM.8	(+) Add, subtract, and multiply matrices of appropriate dimensions.
	H.N-VM.9	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
	H.N-VM.10	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
	H.N-VM.11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
	H.N-VM.12	(+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

[+] indicates standard is beyond College and Career Ready

Less Complex **More Complex**

Standards labeled with a (+) symbol appear in courses intended for all students, but are considered "beyond College and Career Ready" and will not be assessed.

High School Conceptual Category – Algebra

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Algebra
DOMAIN Seeing Structure in Expressions

High School

Cluster	Standards as written	
Interpret the structure of expressions.	**H.A-SSE.1**	Interpret expressions that represent a quantity in terms of its context.⋆
	H.A-SSE.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
	H.A-SSE.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.*
	H.A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*
Write expressions in equivalent forms to solve problems.	**H.A-SSE.3**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
	H.A-SSE.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
	H.A-SSE.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
	H.A-SSE.3c	Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
	H.A-SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*⋆

⋆ indicates Modeling standard

ENTRY POINTS and ACCESS SKILLS for Algebra
Seeing Structure in Expressions Standards in High School

Less Complex ⟷ **More Complex**

For this domain, both clusters are listed together.

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Seeing Structure in Expressions. (A-SSE)	♦ Respond to materials used to add or subtract known and unknown quantities (e.g., 3 plus how many equals 5?) ♦ Attend visually, auditorially, or tactilely to materials used to add or subtract known and unknown quantities ♦ Track materials used to add or subtract known and unknown quantities ♦ Grasp materials used to add or subtract known and unknown quantities ♦ Use two hands to hold materials used to add or subtract known and unknown quantities ♦ Release materials used to add or subtract known and unknown quantities	♦ Answer yes/no questions about expressions ♦ Match parts of an expression to the following terms: term, factor, coefficient, variable, constant, exponent ♦ Create expressions involving addition and subtraction with numbers and with letters representing unknown numbers in real-life situations (e.g., "number of miles plus seven," can be written as "m + 7") ♦ Demonstrate the concept of inequality and use the corresponding symbol correctly (<, >) ♦ Represent a numerical expression involving addition and/or subtraction that represents a real-world problem using manipulatives, drawings, or technology ♦ Represent multiplication expressions using tables, charts, drawings, manipulatives, or technology (e.g., 5 blocks in each of 3 bins)	♦ Simplify algebraic expressions ♦ Interpret numbers written with the exponent of two (e.g., x^2) ♦ Create expressions involving multiplication with numbers and with letters representing unknown numbers in real-life situations (e.g., "three times the number of marbles in a bag" can be written as "3n") ♦ Simplify numerical expressions involving factors, using Order of Operations to (e.g., 3(7 - 2) = 3 x 5 = 15) ♦ Create a numerical expression given a real-world situation (e.g., 20 cookies shared by 5 people; 20 ÷ 5) ♦ Create an expression given the mathematical terms: term, factor, coefficient, variable, constant, exponent	♦ Create an algebraic expression based on a word problem ♦ Simplify linear and nonlinear expressions containing variables, using knowledge of operations ♦ Find the common factors of 2- and/or 3-digit numbers ♦ Create expressions involving one or more operations with numbers and with letters representing unknown numbers in real-life situations (e.g., The number of students in a class that has 20 boys and x girls) ♦ Explain the components of a numerical expression using mathematical terms ♦ Identify equivalent expressions involving the distributive property (e.g. 3(2 + 7) = (3 X 2) + (3 X 7)) ♦ Use the Order of Operations to evaluate a numerical expression that contains parentheses and up to four operations

ENTRY POINTS and ACCESS SKILLS for Algebra
Seeing Structure in Expressions Standards in High School

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Seeing Structure in Expressions. (A-SSE) (continued)	♦ Move materials used to add or subtract known and unknown quantities ♦ Orient materials used to add or subtract known and unknown quantities ♦ Manipulate objects used to add or subtract known and unknown quantities ♦ Locate objects partially hidden or out of sight needed to add or subtract known and unknown quantities (e.g., remove barrier to expose materials) ♦ Use one object to act on another to add or subtract known and unknown quantities (e.g., use scissors to cut materials to be added or subtracted) ♦ Turn on technology to activate program to add/subtract known and unknown quantities (e.g., turn on technology tool to add/subtract)	♦ Simplify numerical expressions involving addition when there are multiple terms (e.g., (4 + 2) + (3 + 7) = 6 + 10) ♦ Create an arithmetic sequence (e.g., 5, 8, 11, 14…) ♦ Apply associative property to expressions involving addition with positive and negative whole numbers (e.g., 3 + (2 + 4) = (3 + 2) + 4	♦ Apply distributive properties to evaluate expressions involving one or more operations with positive and/or negative whole numbers (e.g., 6 x 8 + 6 x 2 = 6 (8 + 2)) ♦ Match the associative property of multiplication with numerical expressions that exemplify the property (e.g., (6 x 4) x 10 = 6 x (4 x 10))	♦ Apply multiple properties to simplify expressions involving one or more operations with numbers and variables (e.g., 4(w - 3) + 2w = 4w – 12 + 2w = 4w + 2w – 12 = 6w – 12) ♦ Produce equivalent expressions by applying the distributive property to problems involving addition, subtraction, and multiplication (e.g., (10 x a) + (10 x b) = 10 (a + b)) ♦ Distinguish between linear and quadratic expressions ♦ Create a geometric sequence (e.g., 2, 4, 8, 16, 32) ♦ Distinguish between a geometric sequence and an arithmetic sequence *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ACCESS SKILLS (continued) for Algebra
Seeing Structure in Expressions Standards in High School

Less Complex ⟷ **More Complex**

	ACCESS SKILLS The student will:	**ENTRY POINTS The student will:**
Seeing Structure in Expressions. (A-SSE) (continued)	♦ Imitate action used to add and/or subtract known and unknown quantities (e.g., imitate classmate attaching icon to add) ♦ Initiate cause-and-effect response during an adding/subtracting activity (e.g., turn on technology tool to activate addition computer program) ♦ Sustain adding/subtracting activity through response (e.g., using preprogrammed voice-generating device comment) ♦ Gain attention during adding/subtracting activity (e.g., raise hand, vocalize) ♦ Make a request during an activity to add and/or subtract known and unknown quantities (e.g., request a turn) ♦ Choose from an array of two in an adding and/or subtracting activity. (e.g., choose materials to be used in adding activity)	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Algebra
DOMAIN Arithmetic with Polynomials and Rational Expressions

High School Mathematics
Algebra **–**
Arithmetic with Polynomials and Rational Expressions

High School

Cluster	Standards as written	
Perform arithmetic operations on polynomials.	**H.A-APR.1**	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
	H.A-APR.1a	Divide polynomials.
Understand the relationship between zeros and factors of polynomials.	**H.A-APR.2**	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
	H.A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Use polynomial identities to solve problems.	**H.A-APR.4**	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
	H.A-APR.5	(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
Rewrite rational expressions.	**H.A-APR.6**	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
	H.A-APR.7	(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

[+] indicates standard is beyond College and Career Ready

ENTRY POINTS and ACCESS SKILLS for Algebra
Arithmetic with Polynomials and Rational Standards in High School

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Arithmetic with Polynomials and Rational Expressions. (A-APR)	♦ Respond to materials used to add and subtract known and unknown quantities ♦ Attend visually, auditorially, or tactilely to materials used to add and subtract known and unknown quantities ♦ Track materials used to add and subtract known and unknown quantities ♦ Shift focus from materials used to add and subtract known and unknown quantities to speaker ♦ Grasp materials used to add and subtract known and unknown quantities ♦ Use two hands to hold materials used to add and subtract known and unknown quantities ♦ Release materials used to add and subtract known and unknown quantities	♦ Classify numbers by the sets to which they belong (e.g., 4/7 is a rational number, but not an integer) ♦ Divide up to a two-digit number by a one-digit number with remainders ♦ Illustrate the concept of "combining like terms" in a simple addition problem using visuals or manipulatives ♦ Add and/or subtract two binomials (e.g., (2x + 4) - (5x + 7))	♦ Add and/or subtract two binomials with two variables (e.g., (2x + 4y) - (5x + 7y)) ♦ Divide up to a three-digit number by a one-digit number with remainders ♦ Interpret the meaning of a remainder of an applied division problem (e.g., How many cars are needed to transport 20 students if each car can hold 6 students?)	♦ Divide up to a three-digit number by a two-digit number with remainders ♦ Add and/or subtract two trinomials (e.g., $(2x^2 + 4x + 4) + (7x^2 + 5x + 3)$) ♦ Multiply two binomials (e.g., (x + 4)(x + 7)) using FOIL strategy (First, Outer, Inner, Last) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ACCESS SKILLS (continued) for Algebra
Arithmetic with Polynomials and Rational Standards in High School

Less Complex ⟷ **More Complex**

	ACCESS SKILLS **The student will:**	**ENTRY POINTS** **The student will:**
Arithmetic with Polynomials and Rational Expressions. (A-APR) (continued)	♦ Move materials used to add and subtract known and unknown quantities ♦ Orient materials used to add and subtract known and unknown quantities ♦ Manipulate objects used to add and subtract known and unknown quantities ♦ Locate objects partially hidden or out of sight needed to add and subtract known and unknown quantities (e.g., remove barrier to expose materials) ♦ Use one object to act on another to add and subtract known and unknown quantities (e.g., use scissors to cut materials) ♦ Turn on technology used to add and subtract known and unknown quantities (e.g., turn on technology tool to add and subtract) ♦ Imitate action used to add and subtract known and unknown quantities (e.g., imitate classmate attaching icon to add) ♦ Initiate cause-and-effect response during an adding/subtracting activity (e.g., turn on technology tool to activate addition computer program) ♦ Sustain adding/subtracting activity through response (e.g., using preprogrammed voice-generating device comment) ♦ Gain attention during adding/subtracting activity (e.g., raise hand, vocalize) ♦ Make a request during an activity to add and subtract known and unknown quantities (e.g., request a turn) ♦ Choose from an array of two in an adding and subtracting activity. (e.g., choose materials to be used in adding activity)	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Algebra
DOMAIN Creating Equations

High School

Cluster	Standards as written	
Create equations that describe numbers or relationships.	**H.A-CED.1**	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. ⋆
	H.A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
	H.A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★
	H.A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* ★

⋆ indicates Modeling standard

ENTRY POINTS and ACCESS SKILLS for Algebra
Creating Equations Standards in High School

Less Complex ⟷ **More Complex**

	ACCESS SKILLS	**ENTRY POINTS**		
	The student will:	**The student will:**	**The student will:**	**The student will:**
Creating Equations. (A-CED)	*See previous algebra cluster for access skills*	♦ Create an equation from a word problem involving addition and/or subtraction ♦ Express an inequality from a word problem involving addition and/or subtraction using symbols (>, <, =) or terms ("is greater than," "is less than," or "is equal to") ♦ Visually represent a real-world problem using manipulatives or technology ♦ Compare number quantities using symbols (>, <, =) or terms ("is greater than," "is less than," or "is equal to") ♦ Express an inequality to represent a real-world problem in one variable (e.g., I have 10 marbles, Tom has more marbles. Represent Tom's number of marbles; t > 10)	♦ Create an equation from a word problem involving ratio or rate (e.g., if there are 12 cookies for 4 students, how many cookies does each student receive?) ♦ Create an equation from a real-life problem involving equivalent rates (e.g., Sam runs 2 laps in 15 minutes. How many laps does he run in 30 minutes?) ♦ Create an equation with a variable to represent a real-world problem involving addition and/or subtraction ♦ Express an inequality involving addition and/or subtraction with a variable using symbols (>, <, =) or terms ("is greater than," "is less than," or "is equal to") ♦ Determine the slope and y-intercept for a line on a coordinate plane ♦ Match an equation to a graph of a line on a coordinate plane ♦ Replace variables with given values in formulas (e.g., Area of a rectangle = Length x Width)	♦ Find equivalent ratios to a ratio given in a real-life problem using manipulatives ♦ Determine the equation for a line on a coordinate plane ♦ Create an equation with a variable to represent a real-world problem involving multiplication and/or division ♦ Express an inequality involving multiplication and/or division with a variable using symbols (>, <, =) or terms ("is greater than," "is less than," or "is equal to") ♦ Graph a line on a coordinate plane to represent the solutions to an equation involving variables ♦ Graph a line in the form y = mx + b ♦ Rearrange formulas to determine different aspects of the formula (e.g., Length = Area/Width) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Algebra
DOMAIN Reasoning with Equations and Inequalities

High School

Cluster	Standards as written	
Understand solving equations as a process of reasoning and explain the reasoning.	**H.A-REI.1**	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
	H.A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Solve equations and inequalities in one variable.	**H.A-REI.3**	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
	H.A-REI.MA3a	Solve linear equations and inequalities in one variable involving absolute value.
	H.A-REI.4	Solve quadratic equations in one variable.
	H.A-REI.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
	H.A-REI.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
	H.A-REI.MA4c	Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations.
Solve systems of equations.	**H.A-REI.5**	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
	H.A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
	H.A-REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

	H.A-REI.8	(+) Represent a system of linear equations as a single matrix equation in a vector variable.
	H.A-REI.9	(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).
Represent and solve equations and inequalities graphically.	**H.A-REI.10**	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
	H.A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.⋆
	H.A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

⋆ indicates Modeling standard
+ indicates standard is beyond College and Career Ready

ENTRY POINTS and ACCESS SKILLS for Algebra
Reasoning with Equations and Inequalities Standards in High School

Less Complex ⟵ ⟶ **More Complex**

	ACCESS SKILLS		**ENTRY POINTS**	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Reasoning with Equations and Inequalities. (A-REI)	♦ Respond to materials used to add and subtract known and unknown quantities ♦ Attend visually, auditorially, or tactilely to materials used to add and subtract known and unknown quantities ♦ Track materials used to add and subtract known and unknown quantities ♦ Shift focus from materials used to add and subtract known and unknown quantities to speaker ♦ Grasp materials used to add and subtract known and unknown quantities ♦ Use two hands to hold materials used to add and subtract known and unknown quantities ♦ Release materials used to add and subtract known and unknown quantities	♦ Identify the value of an unknown variable in an equation (e.g., $2x = 4$) ♦ Evaluate expressions involving addition and subtraction with variables given the value of the variable ♦ Create a line by plotting and connecting two points on a coordinate plane ♦ Identify the solution to an inequality (e.g., $2x < 4$) ♦ Represent the solution to an inequality on a number line ♦ Show the distance from zero (absolute value) for positive and negative numbers (e.g., $\|-7\| = 7$ because -7 is 7 units from zero) ♦ Solve simple equations involving absolute value (e.g., $6 + \|-3\| = x$) ♦ Use properties of equality (applying the same operation to both sides of an equation and canceling out) to simplify equations ♦ Determine the initial value (y-intercept) from a line graphed on a coordinate plane	♦ Represent real situations and mathematical relationships on a coordinate plane ♦ Determine the initial value (y-intercept) and the rate of change (slope) from graphs ♦ Solve equations involving absolute value (e.g., $6 + \|x\| = 9$) ♦ Graph a line on a coordinate plane, given the initial value (y-intercept) and the rate of change (slope) ♦ Graph one or more lines on a coordinate plane given the equation of the lines ♦ Add, subtract, and/or multiply numbers and variables to solve linear equations ♦ Determine the initial value (y-intercept) from an equation in the form of $y = mx + b$ ♦ Determine the rate of change (slope) from an equation in the form of $y = mx + b$	♦ Find the intersection of two lines by graphing them on a coordinate plane ♦ Solve more complex equations involving absolute value (e.g., $6 + \|2x + 1\| = 10$) ♦ Solve linear equations containing variables, using knowledge of properties of equality ♦ Determine the equation of a line represented on a coordinate plane *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS and ACCESS SKILLS for Algebra
Reasoning with Equations and Inequalities Standards in High School

Less Complex ⟷ **More Complex**

	ACCESS SKILLS		ENTRY POINTS	
	The student will:	**The student will:**	**The student will:**	**The student will:**
Reasoning with Equations and Inequalities. (A-REI) (continued)	♦ Move materials used to add and subtract known and unknown quantities ♦ Orient materials used to add and subtract known and unknown quantities ♦ Manipulate objects used to add and subtract known and unknown quantities ♦ Locate objects partially hidden or out of sight needed to add and subtract known and unknown quantities (e.g., remove barrier to expose materials) ♦ Use one object to act on another to add and subtract known and unknown quantities (e.g., use scissors to cut materials) ♦ Turn on technology used to add and subtract known and unknown quantities (e.g., turn on technology tool to add and subtract)	♦ Determine the rate of change (slope) from a line graphed on a coordinate plane		

ACCESS SKILLS (continued) for Algebra
Reasoning with Equations and Inequalities Standards in High School

Less Complex ⟷ **More Complex**

	ACCESS SKILLS **The student will:**	**ENTRY POINTS** **The student will:**
Reasoning with Equations and Inequalities. (A-REI) (continued)	♦ Imitate action used to add and subtract known and unknown quantities (e.g., imitate classmate attaching icon to add) ♦ Initiate cause-and-effect response during an adding/subtracting activity (e.g., turn on technology tool to activate addition computer program) ♦ Sustain adding/subtracting activity through response (e.g., using preprogrammed voice-generating device comment) ♦ Gain attention during adding/subtracting activity (e.g., raise hand, vocalize) ♦ Make a request during an activity to add and subtract known and unknown quantities (e.g., request a turn) ♦ Choose from an array of two in an adding and subtracting activity (e.g., choose materials to be used in adding activity)	

High School Conceptual Category – Functions

	Standards	Entry Points	Access Skills
Interpreting Functions	Pages 195 – 196	Pages 197 – 199	
Building Functions	Pages 200 – 201	Page 202	
Linear, Quadratic, and Exponential Models	Page 203	Pages 204 – 205	
Trigonometric Functions	Page 206	Page 207	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Functions
DOMAIN Interpreting Functions

High School

Cluster	Standards as written	
Understand the concept of a function and use function notation.	**H.F-IF.1**	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
	H.F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
	H.F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n + 1) = f(n) + f(n – 1)$ *for* $n \geq 1$.
Interpret functions that arise in applications in terms of the context.	**H.F-IF.4**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ⋆
	H.F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ⋆
	H.F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ⋆
Analyze functions using different representations.	**H.F-IF.7**	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ⋆
	H.F-IF.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima. ⋆
	H.F-IF.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ⋆
	H.F-IF.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ⋆

⋆ indicates Modeling standard

H.F-IF.7d	(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ⋆
H.F-IF.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ⋆
H.F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
H.F-IF.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
H.F-IF.8b	Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, *and* $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*
H.F-IF.MA 8c	Translate among different representations of functions and relations: graphs, equations, point sets, and tables.
H.F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
H.F-IF.MA 10	Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.

⋆ indicates Modeling standard
+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Functions
Interpreting Functions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Understand the concept of a function and use function notation. (F-IF)	♦ Create an input-output table when given the function rule ♦ Identify the set of numbers to include as possible input values (domain) in a real-life problem that includes non-negative integers (e.g., for the total cost of apples at 50 cents each, the domain for the number of apples is the set of whole numbers) ♦ Identify the set of numbers to include as possible output values (range) in a real-life problem that includes non-negative integers (e.g., if each student has two pencils, the range for the total number of pencils is the set of even numbers from 0 to two times the total number of students) ♦ Label the dependent and independent variable in real-life situation (e.g., the number of students going on a trip (independent) and the number of buses needed (dependent)) ♦ Determine whether the relationship in a given set of ordered pairs or table, involving a domain (input) and range (output) represents a function; i.e., whether a unique output exists for each input (e.g., is this set of ordered pairs a function (3,4) (5,12) (3,8)?) *See entry points for earlier grades in this domain*	♦ Identify the set of numbers to include as possible input values (domain) in a real-life problem that includes real numbers (e.g., if ribbon costs a dollar per yard, the domain for the total yardage of ribbon is the set of non-negative real numbers) ♦ Identify the set of numbers to include as possible output values (range) in a real-life problem that includes real numbers (e.g., if temperature drops 2 degrees per hour for ten hours, the range for the total drop in temperature is the set of real numbers between 0 and -20) ♦ Solve a problem involving dependent and independent variables in a real-life situation (e.g., determine the number of buses needed given the number of students going on a trip) ♦ Determine whether the relationship in a given input-output table or mapping represents a function; i.e., whether a unique output exists for each input ♦ Create a table to extend a simple arithmetic sequence	♦ Determine whether the relationship shown in a graph represents a function; i.e., whether a vertical line drawn through any part of the graph intersects with no more than one point (vertical line test) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Functions
Interpreting Functions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand the concept of a function and use function notation. (continued) (F-IF)	♦ Evaluate expressions by substituting the unknown quantity with more than one given value in mathematical expressions (e.g., find 18x, if x = 2, 3, or 4) ♦ Create an arithmetic sequence using manipulatives (e.g., add a constant number of objects) ♦ Create a relationship showing 1:1 correspondence (e.g., for each object (paper) add one corresponding object (pen)) *See entry points for earlier grades in this domain*		
Interpret functions that arise in applications in terms of the context. (F-IF)	♦ Determine whether the slope of a graph of a linear function is positive or negative ♦ Represent mathematical relationships on a table, graph, or using words, and/or symbols ♦ Produce and interpret tables of ordered pairs that represent the relationship between two variables in everyday situations *See entry points for earlier grades in this domain*	♦ Determine the initial value (y-intercept) from the graph of a function ♦ Determine the rate of change (slope) from the graph of a function ♦ Determine the initial value (y-intercept) of a function from a table of ordered pairs ♦ Determine the rate of change (slope) of a function from a table of ordered pairs ♦ Produce and interpret coordinate graphs based on tables of ordered pairs that represent the relationship between two variables in everyday situations ♦ Determine the rate of change (slope) between two points on a coordinate graph of a linear function	♦ Identify graphs of the following mathematical functions: linear, quadratic, and exponential ♦ Determine the rate of change (slope) between two points on a coordinate graph of a function *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Functions
Interpreting Functions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Analyze functions using different representations. (F-IF)	♦ Graph a function on a coordinate plane given a table of ordered pairs ♦ Compare the rates of change for two functions presented in different ways (e.g., compare the rates of change in Ella's and Kevin's savings accounts based on the description and equation below: (description) Ella deposits $20 in the bank to open an account. She puts $10 into the account each month and withdraws nothing. (equation, where s=savings, m=number of months) $s = 4m + 200$ (answer) Ella's rate of change is $10 per month; and Kevin's is $4 per month) ♦ Describe a graph of a (piecewise) function that has labeled sections (e.g., describe that Part A rises gently, Part B is short but flat, Part C rises like part A, but is steeper, etc.) *See entry points for earlier grades in this domain*	♦ Create a table of ordered pairs based on a linear function (e.g., $f(x) = 2x – 4$) ♦ Create a table of ordered pairs based on a quadratic function (e.g., $f(x) = x^2 - 3$) ♦ Create a table of ordered pairs based on an exponential function (e.g., $f(x) = 2^x + 1$) ♦ Compare initial values of two functions presented in different ways (e.g., $f(x) = 3x – 5$; initial value is -5, and the set of ordered pairs (0,2), (1,0), (2,-2); initial value is 2) ♦ Match a graph of a function with its equation ♦ Match an equation of a function with a table of ordered pairs ♦ Match a table of ordered pairs with its graph	♦ Graph a function on a coordinate plane given its equation ♦ Sketch a (piecewise) graph based on a real-life situation (e.g., Jenny walks at a constant rate to the library, studies for an hour, then runs home. x-axis represents time; y-axis represents distance traveled) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Functions
DOMAIN Building Functions

High School

Cluster	Standards as written	
Build a function that models a relationship between two quantities.	**H.F-BF.1**	Write a function that describes a relationship between two quantities.⋆
	H.F-BF.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.⋆
	H.F-BF.1b	Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* ⋆
	H.F-BF.1c	(+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* ⋆
	H.F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ⋆
Build new functions from existing functions.	**H.F-BF.3**	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
	H.F-BF.4	Find inverse functions.
	H.F-BF.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x – 1)$ for $x \neq 1$.*
	H.F-BF.4b	(+) Verify by composition that one function is the inverse of another.
	H.F-BF.4c	(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
	H.F-BF.4d	(+) Produce an invertible function from a non-invertible function by restricting the domain.

⋆ indicates Modeling standard
+ indicates standard is beyond College and Career Ready

H.F-BF.5	(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

[+] indicates standard is beyond College and Career Ready

ENTRY POINTS for Functions
Building Functions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Build a function that models a relationship between two quantities. (F-BF)	♦ Find the missing value in a table of ordered pairs ♦ Find the missing value within a given arithmetic sequence (e.g., 12, 7, 2, ?, -8 ...) *See entry points for earlier grades in this domain*	♦ Create a function rule (i.e., expression) that represents the values in a table of ordered pairs ♦ Find the missing value within a given geometric sequence (e.g., 2, 6, ?, 54, 162 ...) ♦ Find the specified term outside of a given arithmetic sequence (e.g., 12, 7, 2, -3, -8 ... what is the 12^{th} term?)	♦ Extend a sequence of ordered pairs in a table based on a function rule ♦ Find the specified term outside of a given geometric sequence (e.g., 2, 6, 18, 54, 162 ... what is the 8^{th} term?) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Build new functions from existing functions. (F-BF)			♦ Identify the graph of the inverse of a function *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Functions
DOMAIN Linear, Quadratic, and Exponential Models

High School

Cluster	Standards as written	
Construct and compare linear, quadratic, and exponential models and solve problems.	**H.F-LE.1**	Distinguish between situations that can be modeled with linear functions and with exponential functions. ⋆
	H.F-LE.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ⋆
	H.F-LE.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ⋆
	H.F-LE.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ⋆
	H.F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ⋆
	H.F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ⋆
	H.F-LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ⋆
Interpret expressions for functions in terms of the situation they model.	**H.F-LE.5**	Interpret the parameters in a linear or exponential function in terms of a context. ⋆

⋆ indicates Modeling standard

ENTRY POINTS for Functions
Linear, Quadratic, and Exponential Models Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Construct and compare linear, quadratic, and exponential models and solve problems. (F-LE)	♦ Represent real-life situations and mathematical relationships that can be modeled by a linear function on a table, graph, or using words, and/or symbols *See entry points for earlier grades in this domain*	♦ Interpret tables that represent real-life situations and mathematical relationships that can be modeled by a linear function on a table, graph, or using words, and/or symbols ♦ Represent real-life situations and mathematical relationships that can be modeled by a exponential function on a table, graph, or using words, and/or symbols ♦ Distinguish between linear and non-linear models based on tables representing them ♦ Construct a function that represents the values in a table of ordered pairs	♦ Distinguish between a linear function and a non-linear function based on a real-life situation (e.g., bank account growth from deposits vs. growth from interest) ♦ Distinguish between exponential and non-exponential models based on tables representing them ♦ Interpret tables that represent real-life situations and mathematical relationships that can be modeled by an exponential function on a table, graph, or using words, and/or symbols *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Interpret expressions for functions in terms of the situation they model. (F-LE)	♦ Solve simple linear equations expressed as a real-life situation (e.g., Dan can grow 3 plants from a seed packet. How many total plants can Dan grow from 4 packets?) ♦ Identify the parameters of a given linear function that models a real-life situation (e.g., Dan can grow 3 plants from a seed packet. How many total plants can Dan grow from 4 packets? $f(x) = 3x$; 0 is the initial value and 3 is the rate of change)	♦ Create a table of ordered pairs based on a mathematical problem in a real-life situation (e.g., Dan can grow 3 plants from a seed packet. How many total plants can Dan grow from 4 packets? (0,0) (1,3) (2,6) (3,9) (4,12)) ♦ Solve simple exponential equations expressed as a real-life situation (e.g., Laura opens a bank account with $100 that increases at a rate of 2% per year. How much will be in the account after six years?)	♦ Create a linear equation based on a real-life mathematical problem (e.g., Dan can grow 3 plants from a seed packet. How many total plants can Dan grow from 4 packets? $f(x) = 3x$) ♦ Create a simple exponential equation based on a real-life mathematical problem (e.g., Laura opens a bank account with $100 that increases at a rate of 2% per year. How much will be in the account after six years? $f(x) = 100(1.02)^6$) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Interpret expressions for functions in terms of the situation they model. (continued) (F-LE)	♦ Identify the parameters of a given exponential function that models a real-life situation (e.g., Laura opens a bank account with $100 that increases at a rate of 2% per year. How much will be in the account after six years? $f(x) = 100(1.02)^6$; 100 is the initial value, 1.02 is the growth rate, 6 is the number of years) *See entry points for earlier grades in this domain*		

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Functions
DOMAIN Trigonometric Functions

High School Mathematics *Functions – Trigonometric Functions*

High School

Cluster	Standards as written	
Extend the domain of trigonometric functions using the unit circle.	**H.F-TF.1**	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
	H.F-TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
	H.F-TF.3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
	H.F-TF.4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Model periodic phenomena with trigonometric functions.	**H.F-TF.5**	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.⋆
	H.F-TF.6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
	H.F-TF.7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ⋆
Prove and apply trigonometric identities.	**H.F-TF.8**	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant.
	H.F-TF.9	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

⋆ indicates Modeling standard
+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Functions
Trigonometric Functions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Extend the domain of trigonometric functions using the unit circle. (F-TF)	Standards are considered "beyond College and Career Ready" and will not be assessed.		
Model periodic phenomena with trigonometric functions. (F-TF)	Standards are considered "beyond College and Career Ready" and will not be assessed.		
Prove and apply trigonometric identities. (F-TF)	Standards are considered "beyond College and Career Ready" and will not be assessed.		

High School Conceptual Category – Geometry

	Standards	Entry Points	Access Skills
Congruence	Pages 209 – 210	Pages 211 – 213	
Similarity, Right Triangles, and Trigonometry	Pages 214 – 215	Pages 216 – 217	
Circles	Page 218	Page 219	
Expressing Geometric Properties with Equations	Page 220	Page 221	
Geometric Measurement and Dimension	Page 222	Page 223	
Modeling with Geometry	Page 224	Page 225	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Congruence

High School

Cluster	Standards as written	
Experiment with transformations in the plane.	**H.G-CO.1**	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
	H.G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
	H.G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
	H.G-CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
	H.G-CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Understand congruence in terms of rigid motions.	**H.G-CO.6**	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
	H.G-CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
	H.G-CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Prove geometric theorems.	**H.G-CO.9**	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
	H.G-CO.10	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
	H.G-CO.11	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

	H.G-CO.MA 11a	Prove theorems about polygons. *Theorems include: measures of interior and exterior angles, properties of inscribed polygons.*
Make geometric constructions.	**H.G-CO.12**	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
	H.G-CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

ENTRY POINTS for Geometry
Congruence Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Experiment with transformations in the plane.	♦ On a coordinate plane translate a figure a number of units up or down and left or right ♦ Label a reflection or translation of a two dimensional figure ♦ Demonstrate a reflection or translation of a two-dimensional figure using manipulatives ♦ Show congruence of two figures (e.g., using technology or by tracing/placing a transparency over two figures to show that they are the same) ♦ Describe and identify shapes by attributes, including sizes, angles, and sides ♦ Rotate a figure and draw or identify the result ♦ Identify sides and angles on a figure ♦ Match symmetrical figures ♦ Match concepts to definitions for angle, circle, perpendicular line, parallel line, and/or line segment. *See entry points for earlier grades in this domain*	♦ Determine whether triangles are congruent given their side measures, angles, and shape ♦ Label a rotation of a two-dimensional figure ♦ Demonstrate reflection, translation, and rotation of a two-dimensional figure using manipulatives ♦ Perform a transformation on a figure (e.g., translation, reflection, or rotation) ♦ Find the missing angle measure when one angle measure is given in a diagram showing two parallel lines cut by a transversal, using knowledge of supplementary angles ♦ Identify definitions for angle, circle, perpendicular line, parallel line, and/or line segment ♦ Demonstrate a reflection, translation, or rotation of a two-dimensional figure on a coordinate plane using manipulatives and/or technology ♦ Determine if two shapes are congruent by measuring sides and angles ♦ Describe a variety of transformations on figures, including translation, reflection, rotation, and scale factors ♦ Match symmetrical figures that have been rotated ♦ Perform a 90 degree rotation of a figure on the coordinate plane	♦ Demonstrate a sequence that exhibits the similarity between two given figures on a coordinate plane ♦ Predict the outcome of a transformation on a figure (e.g., translation, reflection, or rotation) ♦ Identify types of symmetry within one figure ♦ Describe figures using properties of sides and angles ♦ Record movement resulting from reflections, rotations, or translations by counting to measure how many spaces each of the vertices of a figure moved left, right, up, or down on a coordinate plane ♦ Predict the outcome of a 90 degree rotation of a figure on the coordinate plane ♦ Distinguish between reflection, translation, and rotation of a two-dimensional figure using manipulatives *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Geometry
Congruence Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand congruence in terms of rigid motions.	♦ Define the terms SSS, SAS, ASA ♦ Match concepts to definitions for SSS, SAS, ASA *See entry points for earlier grades in this domain*	♦ Identify shapes that are congruent* using SSS, SAS, ASA ♦ Decide which theorem (SSS, SAS, ASA) is used to determine if two triangles are congruent) * **Congruent:** A figure is *congruent* with another figure when both have the same size and shape.	♦ Prove that two triangles are congruent, given SSS, SAS, and ASA ♦ Determine side length or angle measure, given congruent triangles *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Prove geometric theorems.	♦ Match theorems to lines, angles, and figures ♦ Describe theorems (e.g., triangle sum, corresponding angle, alternate angle, interior angle theorems) ♦ Label congruent sides and angles within a parallelogram ♦ Identify parallel lines ♦ Draw a transversal through two parallel lines ♦ Identify vertical angles ♦ Determine the perpendicular bisector of a line segment *See entry points for earlier grades in this domain*	♦ Identify an appropriate sequence to solve a problem based on a geometric theorem ♦ Find the sum of the interior angles of triangles using manipulatives or technology ♦ Identify congruent sides and angles within a parallelogram ♦ Find a missing angle measure using the Triangle Sum Theorem ♦ Identify congruent angles created by a transversal through a set of parallel lines ♦ Find the medians of a triangle	♦ Apply geometric theorems to find the measures of the exterior angles of various shapes using manipulatives or technology ♦ Apply geometric theorems to find the measures of the interior angles of various shapes using manipulatives or technology ♦ Determine if sides are congruent ♦ Given angles and side lengths, determine if a figure is a parallelogram ♦ Apply geometric theorems to identify congruent sides and angles within a triangle ♦ Apply theorems to solve for unknown lines and angles in real-world situations ♦ Locate the center of a triangle by determining the intersection of the perpendicular bisectors of its sides *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Geometry
Congruence Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Make geometric constructions.	*Using a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc):* ♦ Identify figures and parts of figures that were created using tools (e.g., compass and straightedge) ♦ Draw two-dimensional geometric figures *See entry points for earlier grades in this domain*	*Using a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc):* ♦ Copy an angle ♦ Copy a line segment ♦ Construct a perpendicular line ♦ Construct parallel lines	*Using a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc):* ♦ Construct a line parallel to a given line through a point not on the line ♦ Bisect an angle ♦ Construct an equilateral triangle so that each vertex of the equilateral triangle is on the circle ♦ Construct a square so that each vertex of the square is on the circle ♦ Construct a regular hexagon so that each vertex of the regular hexagon is on the circle *Continue to address skills and concepts that approach grade-level expectations in this subject*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Similarity, Right Triangles, and Trigonometry

High School

Cluster	Standards as written	
Understand similarity in terms of similarity transformations.	**H.G-SRT.1**	Verify experimentally the properties of dilations given by a center and a scale factor: (1a, 1b)
	H.G-SRT.1a	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
	H.G-SRT.1b	The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
	H.G-SRT.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
	H.G-SRT.3	Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.
Prove theorems involving similarity.	**H.G-SRT.4**	Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
	H.G-SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Define trigonometric ratios and solve problems involving right triangles.	**H.G-SRT.6**	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
	H.G-SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.
	H.G-SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ⋆

⋆ indicates Modeling standard

Apply trigonometry to general triangles.	**H.G-SRT.9**	(+) Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
	H.G-SRT.10	(+) Prove the Laws of Sines and Cosines and use them to solve problems.
	H.G-SRT.11	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

[+] indicates standard is beyond College and Career Ready

ENTRY POINTS for Geometry
Similarity, Right Triangles, and Trigonometry Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand similarity in terms of similarity transformations.	♦ Identify a dilation ♦ Determine if figures appear similar* ♦ Demonstrate a dilation of a two-dimensional figure using manipulatives and/or technology on a coordinate plane *** Similar:** Multiple figures with the same angle measures and different, but proportional, side lengths are *similar*. *See entry points for earlier grades in this domain*	♦ Find a center of dilation for two similar figures ♦ Given a figure, a center of dilation, and a scale factor, draw the resulting figure ♦ Given the lengths of the sides of a figure, a center of dilation, and a scale factor, determine the lengths of the resulting figure ♦ Find the scale factor given two similar figures and some of their measurements ♦ Describe why two figures are similar using the definition of similarity ♦ Determine if figures are similar (Note: similar means same size angles, but different but proportional side lengths)	♦ Demonstrate a sequence that exhibits the similarity between two given rectangles on a coordinate plane ♦ Given two of the angle measures of two triangles, determine if they are similar *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Prove theorems involving similarity.	♦ Solve a problem involving similar figures with measurements provided *See entry points for earlier grades in this domain*	♦ Apply the Pythagorean Theorem to find the length of the sides or the hypotenuse of a right triangle ♦ Solve problems with similar figures (e.g., find the missing angle measure or missing side length)	♦ Solve real-world problems with similar figures (e.g., find the missing angle measure or missing side length) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

ENTRY POINTS for Geometry
Similarity, Right Triangles, and Trigonometry Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Define trigonometric ratios and solve problems involving right triangles.	♦ Label the components of a right triangle (e.g., hypotenuse, legs, right angle) ♦ Identify triangles (e.g., isosceles, right, equilateral, obtuse) ♦ Identify special right triangles (e.g., 30º-60º-90º; 45º-45º-90º) *See entry points for earlier grades in this domain*	♦ Apply the Pythagorean Theorem to find the length of the hypotenuse or sides of a right triangle ♦ Classify triangles (e.g., isosceles, right, equilateral, obtuse)	♦ Show a relationship between the sum of the areas of the squares formed by the legs of a right triangle and the area of the square formed by its hypotenuse, using models to ♦ Apply the Pythagorean Theorem in real-world situations to find a missing length in a naturally occurring right triangle (e.g., length of a ladder) ♦ Describe triangles (e.g., isosceles, right, equilateral, obtuse) ♦ Solve problems involving side relationships of special right triangles *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Apply trigonometry to general triangles.	Standards are considered "beyond College and Career Ready" and will not be assessed.		

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Circles

High School

Cluster	Standards as written	
Understand and apply theorems about circles.	**H.G-C.1**	Prove that all circles are similar.
	H.G-C.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
	H.G-C.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
	H.G-C.MA3a	Derive the formula for the relationship between the number of sides and sums of the interior and sums of the exterior angles of polygons and apply to the solutions of mathematical and contextual problems.
	H.G-C.4	(+) Construct a tangent line from a point outside a given circle to the circle.
Find arc lengths and areas of sectors of circles.	**H.G-C.5**	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

[+] indicates standard is beyond College and Career Ready

ENTRY POINTS for Geometry
Circles Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand and apply theorems about circles.	♦ Create circles, with a given radius, using a compass ♦ Define vocabulary associated with circles (e.g., radius, diameter, chord, angle, arc) ♦ Identify inscribed and central angles *See entry points for earlier grades in this domain*	♦ Illustrate the relationship between inscribed and central angles ♦ Explain the relationship between the diameter/radius and the circumference of a circle ♦ Create a model illustrating the relationship between the circumference and the diameter/radius of a circle ♦ Determine the sum of the measures of interior angles of a given polygon using the formula: $180°(n-2)$	♦ Solve problems involving the relationship between radius, diameter, and circumference of circles ♦ Solve problems involving the relationship between inscribed and central angles *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Find arc lengths and areas of sectors of circles.	♦ Measure the area of a circle ♦ Determine the circumference of a circle using pi (π) *See entry points for earlier grades in this domain*	♦ Identify the relationship between arc and an angle ♦ Solve problems using the relationship between an arc and an angle	♦ Find arc lengths of circles by applying concepts of radius, diameter, and circumference ♦ Find arc measures, in degrees, of circles by applying concept of central and inscribed angles ♦ Solve problems involving the relationship between the inscribed angle and the arc length *Continue to address skills and concepts that approach grade-level expectations in this subject*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Expressing Geometric Properties with Equations

High School Mathematics
Geometry – Expressing Geometric Properties with Equations

High School

Cluster	Standards as written	
Translate between the geometric description and the equation for a conic section.	**H.G-GPE.1**	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
	H.G-GPE.2	Derive the equation of a parabola given a focus and directrix.
	H.G-GPE.3	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
	H.G-GPE.MA3a	(+) Use equations and graphs of conic sections to model real-world problems. ⋆
Use coordinates to prove simple geometric theorems algebraically.	**H.G-GPE.4**	Use coordinates to prove simple geometric theorems algebraically*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,*$\sqrt{3}$*) lies on the circle centered at the origin and containing the point (0, 2).*
	H.G-GPE.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
	H.G-GPE.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
	H.G-GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ⋆

⋆ indicates Modeling standard
+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Geometry
Expressing Geometric Properties with Equations Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Translate between the geometric description and the equation for a conic section.	♦ Given the radius of a circle graph the circle with the center at the origin on the coordinate plane *See entry points for earlier grades in this domain*	♦ Graph a circle, with center at the origin, given an equation ♦ Determine the equation of a circle, with a center at the origin on the coordinate plane, given the radius ($x^2+y^2=r^2$)	♦ Graph a circle with center (h,k) given the equation $(x-h)^2+(y-k)^2=r^2$ *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Use coordinates to prove simple geometric theorems algebraically.	♦ Determine the distance between two points on a horizontal or vertical number line, including negative numbers ♦ Identify slope as positive or negative ♦ Graph a rectangle given the coordinates of four points ♦ Determine the midpoint of two given points on a number line ♦ Identify coordinates of points on the coordinate plane (in any of the four quadrants) ♦ Illustrate concepts of parallel, perpendicular, midpoint, and slope of a line ♦ Determine the length of horizontal and vertical line segments on a coordinate plane *See entry points for earlier grades in this domain*	♦ Determine the coordinates of the midpoint of a vertical or horizontal line segment on the coordinate plane ♦ Determine the slope of a line on a coordinate plane ♦ Determine the length of a line segment on a coordinate plane using the Pythagorean Theorem	♦ Determine midpoints of line segments on a coordinate plane (either given to or drawn by student) ♦ Determine the perimeter of a polygon on a coordinate plane, using Pythagorean Theorem or the distance formula (e.g., triangle, rectangle) ♦ Determine the area of a polygon on a coordinate plane, using Pythagorean Theorem or the distance formula (e.g., triangle, rectangle) ♦ Determine the slope of a line perpendicular to a given line ♦ Use coordinates to compute distance ♦ Determine the length of a line segment on a coordinate plane using the distance formula ♦ Demonstrate that lines that have the same slope are parallel *Continue to address skills and concepts that approach grade-level expectations in this subject*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Geometric Measurement and Dimension

High School Mathematics *Geometry – Geometric Measurement and Dimension*

High School

Cluster	Standards as written	
Explain volume formulas and use them to solve problems.	**H.G-GMD.1**	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
	H.G-GMD.2	(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
	H.G-GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ⋆
Visualize relationships between two-dimensional and three-dimensional objects.	**H.G-GMD.4**	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

⋆ indicates Modeling standard
+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Geometry
Geometric Measurement and Dimension Standards in High School

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Explain volume formulas and use them to solve problems.	♦ Match formulas to shapes to illustrate how volume can be determined for various shapes (e.g., cone, cylinder, prism, sphere, pyramid) *See entry points for earlier grades in this domain*	♦ Calculate the volume of rectangular prisms given the measures of the length, width, and height ♦ Calculate surface area of prisms and/or pyramids using the formula	♦ Calculate the volume of a sphere, using the formula, in real-life and/or mathematical problems ♦ Calculate the volume of a cone, using the formula, in real-life and/or mathematical problems ♦ Calculate the volume of a cylinder, using the formula, in real-life and/or mathematical problems ♦ Calculate the volume of a pyramid, using the formula, in real-life and/or mathematical problems *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Visualize relationships between two-dimensional and three-dimensional objects.	♦ Create cross-section shape, give a three dimensional figure *See entry points for earlier grades in this domain*	♦ Match the cross sectional representation with its original figure (e.g., circle and sphere) ♦ Match three-dimensional objects generated by the rotation of a two dimensional figure	♦ Identify the cross sectional representation of a three dimensional figure (e.g., circle and sphere) ♦ Identify three-dimensional objects generated by the rotation of a two dimensional figure (e.g., rectangle and cylinder) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Geometry
DOMAIN Modeling with Geometry

High School

Cluster	Standards as written	
Apply geometric concepts in modeling situations.	**H.G-MG.1**	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ⋆
	H.G-MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ⋆
	H.G-MG.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ⋆
	H.G-MG.MA4	Use dimensional analysis for unit conversions to confirm that expressions and equations make sense. ⋆

⋆ indicates Modeling standard

ENTRY POINTS for Geometry
Modeling with Geometry Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Apply geometric concepts in modeling situations.	*See entry points for earlier grades in this domain*	♦ Identify the geometric properties of two dimensional objects found in real life (e.g., because my desk surface has two sets of parallel lines and four right angles, it is a rectangle)	♦ Identify the geometric properties of three dimensional objects found in real life *Continue to address skills and concepts that approach grade-level expectations in this cluster*

High School Conceptual Category – Statistics and Probability

	Standards	Entry Points	Access Skills
Interpreting Categorical and Quantitative Data	Pages 227 – 228	Pages 229 – 230	
Making Inferences and Justifying Conclusions	Page 231	Page 232	
Conditional Probability and the Rules of Probability	Pages 233 – 234	Page 235	
Using Probability to Make Decisions	Page 236	Pages 237 – 238	

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Statistics and Probability
DOMAIN Interpreting Categorical and Quantitative Data

High School

Cluster	Standards as written	
Summarize, represent, and interpret data on a single count or measurement variable.	**H.S-ID.1**	Represent data with plots on the real number line (dot plots, histograms, and box plots).⋆
	H.S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ⋆
	H.S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ⋆
	H.S-ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ⋆
Summarize, represent, and interpret data on two categorical and quantitative variables.	**H.S-ID.5**	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ⋆
	H.S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ⋆
	H.S-ID.6a	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. ⋆
	H.S-ID.6b	Informally assess the fit of a function by plotting and analyzing residuals. ⋆
	H.S-ID.6c	Fit a linear function for a scatter plot that suggests a linear association. ⋆

⋆ indicates Modeling standard

Interpret linear models.	**H.S-ID.7**	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ⋆
	H.S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit. ⋆
	H.S-ID.9	Distinguish between correlation and causation. ⋆

⋆ indicates Modeling standard

ENTRY POINTS for Statistics and Probability
Interpreting Categorical and Quantitative Data Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Summarize, represent, and interpret data on a single count or measurement variable. (S-ID)	♦ Represent a simple set of data on a given graphical representation (e.g., bar graph, circle graph, etc.) ♦ Label the axes in a variety of graphical representations of data ♦ Order a set of numerical data from least to greatest ♦ Identify the minimum and maximum values of a set of data ♦ Calculate the range for a set of data ♦ Identify the quartiles in a box plot *See entry points for earlier grades in this domain*	♦ Identify the most appropriate representation for a set of data based on a real-life situation (e.g., circle graph, bar graph, line plot, histogram, dot plot, box plot, line graph, etc.) ♦ Interpret a simple set of data presented on a data display ♦ Calculate the mean, median, and mode for a set of data ♦ Create a box plot given the minimum, first quartile, median, third quartile, and maximum (five-number summary)	♦ Represent the same set of data in two or more graphical representations (e.g., box plot and dot plot) ♦ Create and interpret a set of data in a data display ♦ Identify the mean, median, mode, range, minimum, and maximum for a set of data ♦ Create the minimum, first quartile, median, third quartile, and maximum given a set of data ♦ Determine which measure of center (mean, median, mode) best represents a given set of data that may or may not include outliers *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Summarize, represent, and interpret data on two categorical and quantitative variables. (S-ID)	♦ Create a survey that asks two categorical questions (e.g., Do you prefer Red Sox or Yankees? Were you born in Massachusetts?) ♦ Draw a line of best fit (trend line) for data graphed on a scatter plot *See entry points for earlier grades in this domain*	♦ Create a two-way frequency table based on responses to a survey that asks two categorical questions ♦ Determine whether points on a scatter plot have a linear association ♦ Represent a set of two-variable data on a scatter plot ♦ Use the line of best fit on a scatter plot of real-life data to predict likely outcomes within the range of the data (e.g., child's height over time)	♦ Answer questions based on a two-way frequency table ♦ Identify trends in a two-way frequency table (e.g., people born in Massachusetts tend to prefer the Red Sox) ♦ Explain the purpose of a trend line and demonstrate its use ♦ Calculate the equation of a line of best fit drawn on a scatter plot ♦ Use the line of best fit on a scatter plot of real-life data to predict likely outcomes (e.g., tree's height over time)

ENTRY POINTS for Statistics and Probability
Interpreting Categorical and Quantitative Data Standards in High School

Less Complex ← → **More Complex**

	The student will:	**The student will:**	**The student will:**
Interpret linear models. (S-ID)	♦ Determine the correlation (positive, negative, or none) of data on a scatter plot ♦ Determine the line of best fit for a set of data using a graphing calculator *See entry points for earlier grades in this domain*	♦ Describe the strength of the correlation of data in a scatter plot using terminology (e.g., "strong positive," "weak negative," "none", etc.) ♦ Interpret the correlation coefficient (r) for a set of data in a scatter plot found by using a graphing calculator	♦ Interpret the line of best fit drawn on a scatter plot in terms of the context of the data ♦ Interpret the relationship between two variables in a scatter plot when there is a linear relationship (e.g., math scores vs. number of days students were absent) ♦ Interpret the correlation of the line of best fit for data in a scatter plot in terms of a context (e.g., based on temperature vs. ice cream sales, do ice cream sales increase as the temperature rises?) ♦ Interpret the y-intercept of the line of best fit for data in a scatter plot in terms of a context (e.g., temperature between noon and 6 p.m.; y-intercept is the temperature at noon) ♦ Interpret the slope of the line of best fit for data in a scatter plot in terms of a context (e.g., for every hour of television watched, test scores decreased by 7 points) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Statistics and Probability
DOMAIN Making Inferences and Justifying Conclusions

High School

Cluster	Standards as written	
Understand and evaluate random processes underlying statistical experiments.	**H.S-IC.1**	Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population. ⋆
	H.S-IC.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	**H.S-IC.3**	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★
	H.S-IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★
	H.S-IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★
	H.S-IC.6	Evaluate reports based on data. ★

⋆ indicates Modeling standard

ENTRY POINTS for Statistics and Probability
Making Inferences and Justifying Conclusions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand and evaluate random processes underlying statistical experiments.	♦ Identify a representative data sample of a given population ♦ Identify situations in which a sample is required in order to analyze a population (e.g., eye color in class vs. eye color in town) *See entry points for earlier grades in this domain*	♦ Apply the terms *population, data sample, validity,* and/or *data set* to describe a real-life situation ♦ Distinguish between different types of samples (e.g., random, systematic, convenience, and cluster) ♦ Identify a random sample of a given population	♦ Identify variables that can affect the outcome of sampling data from a given population ♦ Create different types of samples (e.g., random, systematic, convenience, and cluster) based on a real-life situation ♦ Explain how a random sample could be obtained from a larger population set *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	♦ Draw conclusions given a data sample ♦ Compare two sets of numerical data (e.g., the height of players on the football vs. basketball teams) ♦ Identify which data is important to collect in a given situation *See entry points for earlier grades in this domain*	♦ Predict patterns and/or outcomes based on observations about data ♦ Compare data from a local survey to a parameter (e.g., class distribution of men vs. women to national statistics)	♦ Draw conclusions and/or make predictions based on data gathered (or to be gathered) ♦ Draw conclusions after comparing data from a random sample vs. a non-random sample (e.g., compare the selection of favorite songs of a random sample of the school vs. your best friends) *Continue to address skills and concepts that approach grade-level expectations in this cluster*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Statistics and Probability
DOMAIN Conditional Probability and the Rules of Probability

High School

Cluster	Standards as written	
Understand independence and conditional probability and use them to interpret data.	**H.S-CP.1**	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). ⋆
	H.S-CP.2	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ⋆
	H.S-CP.3	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. ⋆
	H.S-CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* ⋆
	H.S-CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* ⋆

⋆ indicates Modeling standard
+ indicates standard is beyond College and Career Ready

Use the rules of probability to compute probabilities of compound events in a uniform probability model.	**H.S-CP.6**	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. ⋆
	H.S-CP.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. ⋆
	H.S-CP.8	(+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model. ⋆
	H.S-CP.9	(+) Use permutations and combinations to compute probabilities of compound events and solve problems. ⋆

⋆ indicates Modeling standard
+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Statistics and Probability
Conditional Probability and the Rules of Probability Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Understand independence and conditional probability and use them to interpret data.	♦ Identify the probability of a specific outcome ♦ List the possible outcomes (sample space) of an event (e.g., 6 possible outcomes of rolling 1 die; 36 possible outcomes of rolling 2 dice) *See entry points for earlier grades in this domain*	♦ Distinguish between independent and dependent events in a real-life situation (e.g., choosing marbles from a bag with replacement vs. without replacement) ♦ Determine conditional probability based on a given table ♦ Answer questions based on two-way frequency tables of categorical data (e.g., based on two questions asked of the same sample) ♦ Answer questions based on Venn diagrams of categorical data (e.g., based on two questions asked of the same sample)	♦ Determine probabilities based on two-way frequency tables of categorical data (e.g., what is the probability of a person owning a dog and having allergies) ♦ Determine probabilities based on Venn diagrams of categorical data (e.g., what is the probability of an athlete playing on both the football and baseball teams) *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	♦ Create Venn diagrams given categorical data (e.g., 25 students on the field hockey team; 20 students on the softball team; and 6 students on both) ♦ Create two-way frequency tables given categorical data (e.g., favorite pizza topping vs. favorite type of movie) *See entry points for earlier grades in this domain*	♦ Calculate probabilities from Venn diagrams ♦ Calculate probabilities from two-way frequency tables	♦ Calculate probabilities of compound events from Venn diagrams using the Addition Rule (e.g., the probability of A, the probability of B, the probability of A and B, the probability of A or B, the probability of neither A nor B) ♦ Calculate probabilities of compound events from two-way frequency tables using the Addition Rule (e.g., the probability of students who prefer horror or adventure movies who also prefer pepperoni pizza) *Continue to address skills and concepts that approach grade-level expectations in this subject*

CONTENT AREA Mathematics
CONCEPTUAL CATEGORY Statistics and Probability
DOMAIN Using Probability to Make Decisions

High School Mathematics *Statistics and Probability – Using Probability to Make Decisions*

High School

Cluster	Standards as written	
Calculate expected values and use them to solve problems.	**H.S-MD.1**	(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. ⋆
	H.S-MD.2	(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ⋆
	H.S-MD.3	(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* ⋆
	H.S-MD.4	(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* ⋆
Use probability to evaluate outcomes of decisions.	**H.S-MD.5**	(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.⋆
	H.S-MD.5a	(+) Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* ⋆
	H.S-MD.5b	(+) Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* ⋆
	H.S-MD.6	(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ⋆
	H.S-MD.7	(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game[5]) ⋆

[5] Replacing the hockey goalie with an extra skater.
⋆ indicates Modeling standard
+ indicates standard is beyond College and Career Ready

ENTRY POINTS for Statistics and Probability
Using Probability to Make Decisions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	**The student will:**	**The student will:**
Calculate expected values and use them to solve problems.	♦ Determine which factors help to arrive at a likely prediction of an outcome ♦ Determine the possible outcomes of an event or experiment ♦ Define and apply the terms population, data sample, validity, and data set to demonstrate understanding of probability *See entry points for earlier grades in this domain*	♦ Design and/or conduct an experiment in which data is used to make a prediction ♦ Use organized lists, graphs, models, tables, and diagrams to portray data and/or predict outcomes ♦ Record data and make predictions of outcomes for repeated trials ♦ Identify a sample of a given population ♦ Recognize that data samples are taken for a population that is too large to analyze in its entirety	♦ Use tree diagrams, tables, and lists to represent possible or actual outcomes of trials ♦ Use ratios to represent the probability of an outcome of an experiment ♦ Given a population, select different sample sizes and make predictions about validity of results for each sample ♦ Recognize that the data taken for a sample is not exact, and will result in a prediction about the population ♦ Identify the variables that can affect the analysis/outcome of sampling data from a given population ♦ Identify the benefits and drawbacks to sampling data from a population to make analyses and observations *Continue to address skills and concepts that approach grade-level expectations in this cluster*
Use probability to evaluate outcomes of decisions.	♦ Determine which factors help to arrive at a likely prediction of an outcome ♦ Determine the possible outcomes of an event or experiment ♦ Define and apply the terms population, data sample, validity, and data set to demonstrate understanding of probability *See entry points for earlier grades in this domain*	♦ Design and/or conduct an experiment in which data is used to make a prediction ♦ Use organized lists, graphs, models, tables, and diagrams to portray data and/or predict outcomes ♦ Record data and make predictions of outcomes for repeated trials ♦ Identify a sample of a given population ♦ Recognize that data samples are taken for a population that is too large to analyze in its entirety	♦ Use tree diagrams, tables, and lists to represent possible or actual outcomes of trials ♦ Use ratios to represent the probability of an outcome of an experiment ♦ Given a population, select different sample sizes and make predictions about validity of results for each sample ♦ Recognize that the data taken for a sample is not exact, and will result in a prediction about the population

ENTRY POINTS for Statistics and Probability
Using Probability to Make Decisions Standards in High School

Less Complex ⟷ **More Complex**

	The student will:	The student will:	The student will:
Use probability to evaluate outcomes of decisions. (continued)			♦ Identify the variables that can affect the analysis/outcome of sampling data from a given population ♦ Identify the benefits and drawbacks to sampling data from a population to make analyses and observations *Continue to address skills and concepts that approach grade-level expectations in this subject*